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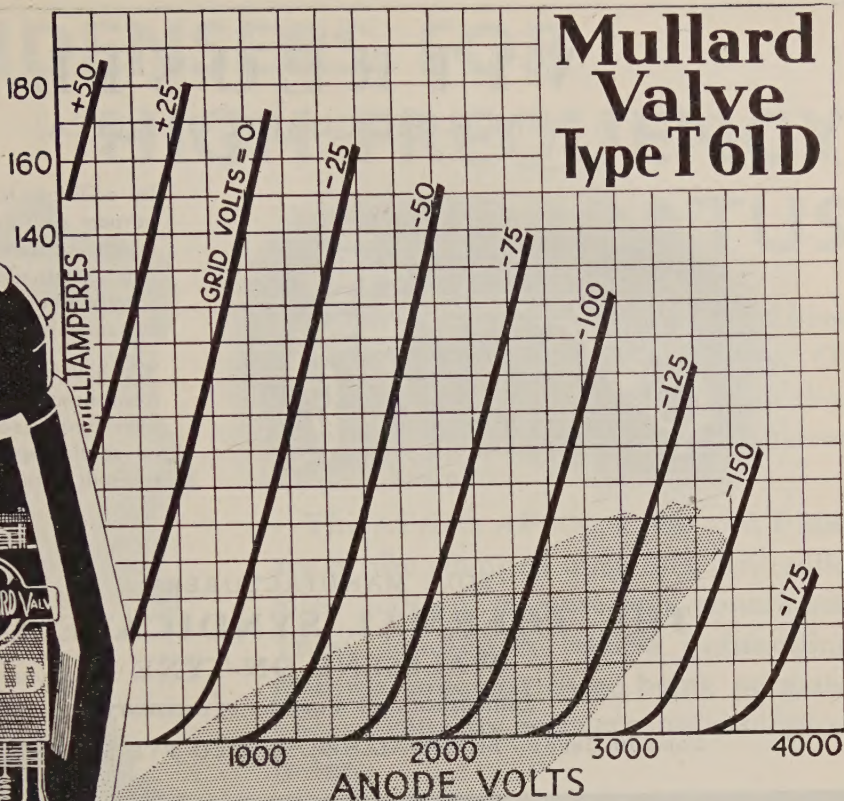
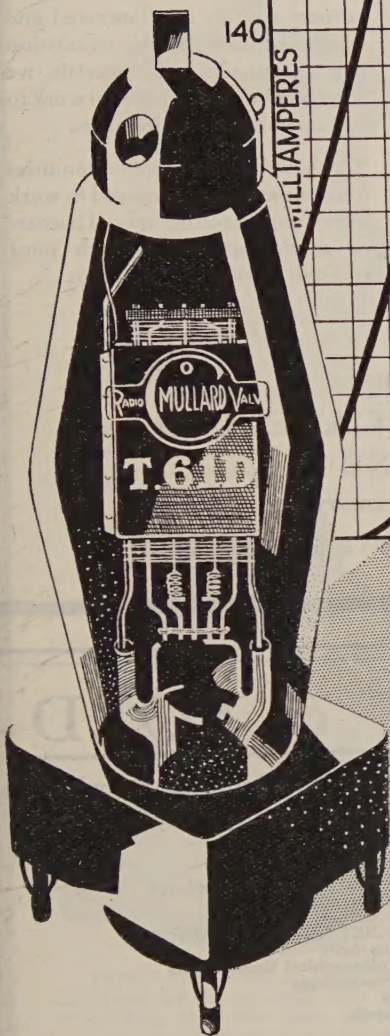
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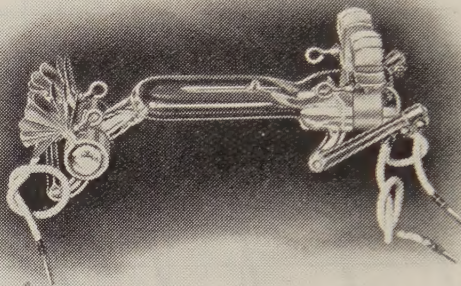
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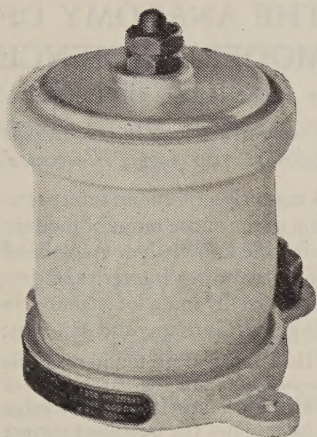
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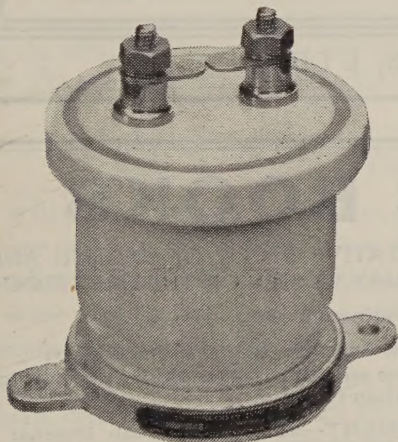
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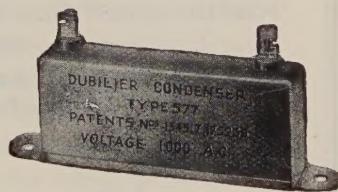
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THE PROCEEDINGS OF THE PHYSICAL SOCIETY

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RELATIONS BETWEEN THE COMBINATION COEFFICIENTS OF ATMOSPHERIC IONS

BY F. J. W. WHIPPLE, SC.D., F.INST.P.

Received December 12, 1932. Read February 3, 1933.

ABSTRACT. The principal object of the paper is to put forward for consideration a formula,

$$\eta_{12} - \eta_{10} = 4\pi ew_1,$$

which indicates that the combination coefficient η_{12} for small ions and large ions of the opposite sign exceeds the coefficient η_{10} for small ions and uncharged nuclei, and further that the difference between the two coefficients depends on the mobility w_1 of the small ions. The experimental evidence for the formula is discussed, as well as possible applications.

§ 1. INTRODUCTION

IN the theory of the conductivity of the atmosphere, rates of combination of ions are of great importance. These rates are difficult to determine and it would be advantageous if the necessary observations could be simplified by the application of some new general principle. As long ago as 1902 Langevin investigated the relation between the rate of combination of ions and their mobilities; the ions in question were those produced by radioactive processes in filtered air. In atmospheric electricity such ions are known as "small ions," to distinguish them from the large ions of small mobility. As a limiting case in his theory Langevin found a formula which would hold if every close approach of two small ions of opposite sign led to a combination. His experimental investigation demonstrated that the formula was not satisfied for air at atmospheric pressure, but it became a good approximation at higher pressures.

By using Langevin's method we can obtain similar formulae involving the combination coefficients which determine the rates of combination of small ions with large ions and with uncharged nuclei. The new formulae appear to be in accordance with experimental data. Though it is true that these data are too scanty to establish the validity of any general theory, the verification is satisfactory as far as it goes.

Some tentative applications of the new formulae have been made. For example the question whether the proportions of charged and uncharged Aitken nuclei in the air vary systematically is discussed. At the end of the paper the variation of the combination coefficients in air which is kept stagnant a long while is considered, and evidence is quoted to show that the combination coefficients for small ions and uncharged nuclei increase in the same proportion as the areas of the individual nuclei.

§ 2. NOTATION

The following notation will be used.

N	N is the number of Aitken nuclei per unit volume;		
N_1	N_1	„	of large + ions (i.e. nuclei with + charges);
N_2	N_2	„	of large - ions (i.e. nuclei with - charges);
N_0	N_0	„	of uncharged nuclei;
n_1	n_1	„	of small + ions;
n_2	n_2	„	of small - ions;
w_1	w_1 the mobility of a small + ion, i.e. the velocity in a field of unit strength in the electrostatic system of units; and		
w_2	w_2 the mobility of a small - ion.		

The combination coefficient η_{10} is defined by the statement that the frequency of combinations of small + ions with uncharged nuclei is $\eta_{10} n_1 N_0$ per unit volume per unit time. Similar definitions hold for η_{12} , η_{20} and η_{21} . The mutual combination coefficient for small ions of opposite signs is α .

It will be assumed that each of the large ions carries the single electronic charge e . This has been verified* by J. J. Nolan, Boylan and de Sachy who found that

$$N = N_0 + N_1 + N_2 \quad \dots\dots(2.1),$$

N and N_0 being determined by the Aitken nucleus counter, whilst N_1 and N_2 were measured electrically on the assumption that the charge on each ion was e .

In the notation adopted here, the formula suggested by Langevin is

$$\alpha = 4\pi e (w_1 + w_2) \quad \dots\dots(2.2).$$

The new formulae which will be examined are

$$\eta_{12} - \eta_{10} = 4\pi e w_1 \quad \dots\dots(2.3),$$

and

$$\eta_{21} - \eta_{20} = 4\pi e w_2 \quad \dots\dots(2.31).$$

§ 3. LANGEVIN'S EQUATION

The argument† leading to Langevin's formula is briefly as follows. If a positive ion is at A and a negative at B and there are no other ions at a distance comparable with that between A and B , then the ions will move straight towards each other. The force acting on each is e^2/AB^2 and the relative velocity is $(w_1 + w_2) e/AB^2$.

* *Proc. R. Irish Acad.* 37 A (1925).

† *Ann. de Chim. et de Phys.* 28, 438 (1903). Cf. Sir J. J. Thomson and G. P. Thomson, *Conduction of Electricity through Gases* (3rd ed. 47, 1928). N.B. The α of the present notation is ae in Langevin's.

Since n_2 is the density of the $-$ ions, the number of these crossing in unit time the surface of a sphere with centre A and radius AB is on the average,

$$4n_2\pi AB^2 (w_1 + w_2) e/AB^2 \text{ or } 4n_2\pi (w_1 + w_2) e.$$

There are n_1 positive ions like A in unit volume, so the number of close approaches in unit time is $4n_1n_2\pi (w_1 + w_2) e$. If every close approach led to combination, then the combination coefficient α would satisfy the proposed relation,

$$\alpha = 4\pi e (w_1 + w_2).$$

The term combination is used rather loosely here. The small ion is supposed to be formed of a chain or cluster of about a dozen molecules associated with an ionized atom of oxygen or nitrogen. Probably when two ions of opposite signs collide the ionized atoms recombine to make a molecule, and the clusters of molecules break up.

This summary of Langevin's argument is too much simplified. Whilst it takes account of the velocity of the ion due to electrical attraction, the velocity due to thermal agitation or Brownian movement is ignored. On account of this agitation the relative velocity of two ions will not in general be along the line joining them; they may approach one another but pass by without collision, each describing an arc of a hyperbola.

Experiments have shown that the formula is not valid for ordinary pressures. Langevin developed a method by which he could determine ζ , the ratio

$$\alpha/4\pi (w_1 + w_2) e.$$

If the formula had always held, then the value of ζ would have been unity. Actually Langevin obtained* the following results:

Pressure (mm.)	152	375	760	1550	2320	3800
ζ	0.01	0.06	0.27	0.62	0.80	0.90

For low pressures ζ is nearly proportional to the square of the pressure. Since w_1 and w_2 are inversely proportional to the pressure, it is implied that α is proportional to the pressure. Langevin explains this by supposing that a preliminary to combination is a collision of one of the ions with a gas molecule when the ions are within a certain distance. The chance of such a collision is proportional to the pressure. For a pressure of five atmospheres the value of ζ was 0.90. It is likely that for higher pressures the approximation to unity would be much closer. I do not know if the appropriate experiments have been made. None are quoted by Thomson and Thomson. It seems however that at a pressure of, say, 10 atmospheres the air molecules would be so close together that one ion sweeping round another of opposite sign would be certain to collide with air molecules, lose its angular momentum, and be captured. Under such circumstances the formula

$$\alpha = 4\pi e (w_1 + w_2)$$

would be valid.

* *Ann. de Chim. et de Phys.* 28, 483 (1903); *Comptes Rendus*, 137, 177 (1903); cf. Thomson and Thomson, *loc. cit.* p. 35.

§ 4. THE COMBINATION OF SMALL IONS WITH CONDENSATION-NUCLEI CHARGED OR UNCHARGED

The significance of η_{10} . Now let us consider the circumstances in which the combinations of small ions with uncharged nuclei take place. Both ions and nuclei are subject to thermal agitation or Brownian movement, but the nuclei, being comparatively large and heavy, will be moving comparatively slowly. The frequency of collisions will depend on the sizes of the particles and on the velocities of the ions. The electrical attraction between an ion and an uncharged nucleus, being small, will not affect the frequency of collisions, but the attraction probably ensures that the ion adheres to the nucleus and so makes a large ion. We adopt as a working hypothesis the view that the expression $\eta_{10}n_1N_0$, which by definition represents the rate of combination of small ions and neutral nuclei, also represents the frequency of collisions between such bodies.

The significance of η_{12} . In the case of collisions between small ions and nuclei which carry opposite charges the electrical attraction has to be considered as well as the Brownian movement. At a distance r from a large ion with the electronic charge $-e$ the strength of the electric field is er^{-2} . The resulting velocity of a small positive ion with mobility w_1 is accordingly ew_1r^{-2} . This velocity is to be added vectorially to the random Brownian velocity.

Accordingly the number of small positive ions entering in unit time a small sphere of radius r surrounding the charged nucleus exceeds the number which would enter such a sphere surrounding an uncharged nucleus by $ew_1r^{-2}n_14\pi r^2$ or $4\pi ew_1n_1$.

More precisely it may be stated that the number of small positive ions entering in unit time the small spheres surrounding N_2 charged nuclei exceeds the number which would enter such spheres surrounding N_2 uncharged nuclei by $4\pi ew_1n_1N_2$. If the spheres are only just large enough to contain the nuclei it is to be expected that every small ion entering a sphere will be captured. Had the N_2 charged nuclei been uncharged the rate of capture of small ions would have been $\eta_{10}n_1N_2$. Owing to the electrical attraction the rate is increased by $4\pi ew_1n_1N_2$. The total rate is, by definition, $\eta_{12}n_1N_2$. It follows that

$$\eta_{12} = \eta_{10} + 4\pi ew_1 \quad \dots\dots(4.1).$$

Exactly the same argument leads to the analogous formula

$$\eta_{21} = \eta_{20} + 4\pi ew_2 \quad \dots\dots(4.2).$$

It was seen that the similar formula for α did not hold good, unless the molecules of the air were so close together that their distances apart were comparable with the size of a small ion. The new formulae should hold if the spacing of the air molecules is small compared with the size of a nucleus. This condition is satisfied, for at atmospheric pressure the interval between air molecules is of the order 3×10^{-7} cm., whilst the diameter of a nucleus or large ion is about 10^{-5} cm. Thus the case for the new formulae is established, at any rate so far as to encourage an appeal to experiment.

§ 5. THE VERIFICATION OF THE NEW FORMULAE

The formulae to be discussed are

$$\eta_{12} - \eta_{10} = 4\pi e w_1 \dots\dots(5.1)$$

and

$$\eta_{21} - \eta_{20} = 4\pi e w_2 \dots\dots(5.11).$$

Apparently the only observations available for the verification of the formulae are those made in 1925 by Nolan, Boylan and de Sachy. In their investigation they dealt with the air of a laboratory. They had facilities for measuring the rate of production of ionization, the numbers of ions, large and small, and also the number of uncharged nuclei.

Denoting by q the rate of ionization, we can write down four equations by which the combination coefficients and the observed quantities are related. The first two equations indicate that the number of small ions of one sign or the other formed in unit time is equal to the number absorbed by nuclei, charged or uncharged. The other two equations indicate that the number of uncharged nuclei receiving charges is equal to the number of charged nuclei which are neutralized. The four equations are:

$$q = \eta_{12} n_1 N_2 + \eta_{10} n_1 N_0 \dots\dots(5.2),$$

$$q = \eta_{21} n_2 N_1 + \eta_{20} n_2 N_0 \dots\dots(5.21),$$

$$\eta_{10} n_1 N_0 = \eta_{21} n_2 N_1 \dots\dots(5.3),$$

$$\eta_{20} n_2 N_0 = \eta_{12} n_1 N_2 \dots\dots(5.31).$$

The equations imply that nuclei are so much more numerous than small ions, that a small ion is sure to terminate its career by an encounter with a nucleus; the recombination of small ions with each other is ignored. It is also assumed that the combination of small ions with gross particles of dust, particles much larger but much less numerous than condensation nuclei, can be neglected. At first glance it might be supposed that, if n_1 , n_2 , N_0 , N_1 and N_2 were all known as well as q , the four combination coefficients could be deduced. This is not the case however, for the equations are not independent.

In 1925 Nolan, Boylan and de Sachy* measured in the laboratory N_1 and n_2 . The value of q was found at the same time, three separate pieces of apparatus being used. It was assumed that $N_1 = N_2$, that $n_1 = n_2$ and also that $\eta_{21} = \eta_{12}$. These assumptions imply that $\eta_{10} = \eta_{20}$.

The ranges observed were for N_1 from 3300 to 11880 ions per cm^3 ; for n_2 from 105 to 497 ions per cm^3 ; for q from 17.8 to 64.7 ions per cm^3 per second. By using the equation

$$q = 2\eta_{21} n_2 N_1,$$

it was deduced that η_{21} ranged from 6.2×10^{-6} to 13.1×10^{-6} and the average value of η_{21} was given by

$$\eta_{21} = 9.7 \times 10^{-6}.$$

* *Proc. R. Irish Acad.* A 37 (1925).

From observations of the ratio N_0/N_1 the average value 1.28 was deduced and hence it was found that

$$\eta_{20} = \eta_{21}/1.28 = 7.6 \times 10^{-6}.$$

According to the assumptions mentioned above it followed that

$$\eta_{12} = \eta_{21} = 9.7 \times 10^{-6}$$

and

$$\eta_{10} = \eta_{20} = 7.6 \times 10^{-6}.$$

In 1927 Nolan and de Sachy introduced an amendment*. They gave new values of η_{12} and η_{10} whilst keeping the original values of η_{21} and η_{20} . The amendment was based on the observational result, $n_1 n_2 = 1.11$. The equality of N_1 and N_2 was still accepted and a new assumption was made, viz.

$$\eta_{21} : \eta_{20} = \eta_{12} : \eta_{10}.$$

The amended results were

$$\eta_{21} = 9.7 \times 10^{-6}, \quad \eta_{20} = 7.6 \times 10^{-6},$$

$$\eta_{12} = 8.7 \times 10^{-6}, \quad \eta_{10} = 6.8 \times 10^{-6}.$$

These figures have to be compared with those representing the mobilities. For indoor air Nolan and de Sachy found† mobilities 1.19 and 1.38, reckoned in cm. sec. for a gradient of 1 volt/cm. Hence, using electrostatic units, we have

$$4\pi ew_1 = 4\pi \times 4.77 \times 10^{-10} \times 300 \times 1.19 = 2.14 \times 10^{-6},$$

$$4\pi ew_2 = 4\pi \times 4.77 \times 10^{-10} \times 300 \times 1.38 = 2.48 \times 10^{-6}.$$

The formulae under discussion require

$$\eta_{12} - \eta_{10} = 4\pi ew_1, \quad \text{i.e.} \quad 1.9 \times 10^{-6} = 2.14 \times 10^{-6},$$

and

$$\eta_{21} - \eta_{20} = 4\pi ew_2, \quad \text{i.e.} \quad 2.1 \times 10^{-6} = 2.48 \times 10^{-6}.$$

The approximate verification is satisfactory, but confirmation by additional observations is clearly desirable.

§ 6. INDIRECT ESTIMATES OF COMBINATION COEFFICIENTS AND OF q

If we assume the new formulae we can use observations of $w_1, w_2, n_1, n_2, N_1, N_2$ and N_0 to determine not only the η 's but also q . The equations of equilibrium are

$$\eta_{10} n_1 N_0 = (\eta_{20} + 4\pi ew_2) n_2 N_1 \quad \dots\dots(6.1),$$

$$\eta_{20} n_2 N_0 = (\eta_{10} + 4\pi ew_1) n_1 N_2 \quad \dots\dots(6.11).$$

From these it follows that

$$\eta_{10} = \frac{4\pi e N_1 (w_1 n_1 N_2 + w_2 n_2 N_0)}{n_1 (N_0^2 - N_1 N_2)} \quad \dots\dots(6.2),$$

$$\eta_{20} = \frac{4\pi e N_2 (w_2 n_2 N_1 + w_1 n_1 N_0)}{n_2 (N_0^2 - N_1 N_2)} \quad \dots\dots(6.21),$$

and we deduce that

$$q = \frac{4\pi e N_0 [n_1 w_1 N_2 (N_1 + N_0) + n_2 w_2 N_1 (N_2 + N_0)]}{N_0^2 - N_1 N_2} \quad \dots\dots(6.3).$$

* *Proc. R. Irish Acad.* A 37, 71 (1927).

† *Loc. cit.* p. 83.

Let us accept the following data from the Dublin experiments:

$$4\pi ew_1 = 2.14 \times 10^{-6}, \quad 4\pi ew_2 = 2.48 \times 10^{-6}, \quad N_0 = 1.28 N_1 = 1.28 N_2 \text{ and} \\ n_1 = 1.11 n_2.$$

On substitution we find that

$$\eta_{21} = 11.2 \times 10^{-6}, \quad \eta_{12} = 9.9 \times 10^{-6}, \quad \eta_{20} = 8.7 \times 10^{-6}, \quad \eta_{10} = 7.8 \times 10^{-6}.$$

These coefficients, which agree well enough with those given by Nolan and his collaborators, have been determined, it will be noticed, without using absolute values of the N 's and n 's.

The method can be used most readily for finding q under such circumstances that the whole conductivity of the air may be regarded as due to the small ions and that the net charge on the large ions is zero. Then

$$\lambda_1 = n_1 ew_1, \quad \lambda_2 = n_2 ew_2, \quad N_1 = N_2$$

$$\text{and } q = 4\pi (\lambda_1 + \lambda_2) N_0 N_1 / (N_0 - N_1) = 4\pi (\lambda_1 + \lambda_2) N_0 (N - N_0) / (3N_0 - N) \\ \dots\dots(6.31).$$

The difficult counts of large ions are obviated and q can be estimated for the open air by the use of comparatively simple apparatus*.

It is clear however that the method suffers from two disadvantages. In the first place the formula for q is a bad one because, in the determination of a difference like $(3N_0 - N)$, a lower order of accuracy is attained than in the measurements of N_0 and of N . Secondly there is a more fundamental difficulty. In discussions of this subject it is customary to assume that there is local equilibrium between the process of generation of ions and the processes of combination. The turbulent motion of the air, carrying ions from level to level, is ignored, as well as the conductivity current. A complete analysis would be very difficult, but in the absence of that analysis it must be anticipated that our simple formula for determining q will not always lead to consistent results.

§7. THE NUMBERS OF CHARGED AND UNCHARGED NUCLEI AND THE RATE OF DISSIPATION OF SMALL IONS.

Let the rate of dissipation of small ions of positive sign be denoted by $\beta_1 n_1$. β_1
Small ions disappear on account of combination with small ions of opposite sign or on account of combination with nuclei, charged or uncharged, so that

$$\beta_1 n_1 = \alpha n_1 n_2 + \eta_{10} n_1 N_0 + \eta_{12} n_1 N_2 \quad \dots\dots(7.1).$$

* An instrument designed for measurements of N and N_0 , a modification of the Aitken nucleus-counter, was described in 1931 by J. Scholz, *Z. f. Instrumentkunde*, 51, 505 (1931), but the inventor has demonstrated recently that the elimination of the charged nuclei in the receiver is very slow. The method of use which he recommends now suffers from the drawback that the mobility of the large ions has to be given an assumed constant value, *Met. Z.* 49, 388 (1932). At Kew Observatory a separate condenser attached to a "portable" Aitken counter is being used for eliminating the large ions and is giving consistent readings.

In the foregoing discussion it has been assumed that the first term on the right of this equation is negligible in comparison with the other two, so that

$$\beta_1 = \eta_{10}N_0 + \eta_{12}N_2 \quad \dots\dots(7.11).$$

If the formula (2.3) is valid, then

$$\beta_1 = \eta_{10}(N_0 + N_2) + 4\pi ew_1N_2 \quad \dots\dots(7.12).$$

For the sake of simplicity let us discuss the case in which there is complete symmetry as between positive and negative charges. In this case the condition for the steady state may be written

$$\frac{N_0}{1+x} = \frac{N_1}{x} = \frac{N_2}{x} = \frac{N}{1+3x} \quad \dots\dots(7.2),$$

x where

$$x = \eta_{10}/4\pi ew_1 \quad \dots\dots(7.3).$$

Hence it follows that

$$\beta_1 = 4\pi ew_1 \frac{2x(1+x)}{1+3x} N \quad \dots\dots(7.4).$$

It is convenient to write

$$\beta_1 = 4\pi ew_1 y N \quad \dots\dots(7.5),$$

y so that

$$y = 2x(1+x)/(1+3x) \quad \dots\dots(7.51).$$

This relation between x and y is illustrated by figure 1. The variables x and y are proportional to the coefficients which are used to express the rates of combination of

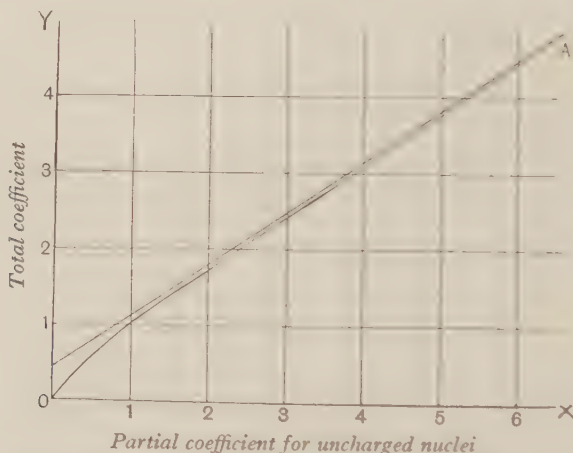


Figure 1. The total combination coefficient for small ions and nuclei as a function of the combination coefficient for small ions and uncharged nuclei.

$$x = \frac{\eta_{10}}{4\pi ew_1}, \quad y = \frac{\beta_1}{4\pi ew_1 N}.$$

The functional relation

$$y = \frac{2x(1+x)}{(1+3x)}$$

is represented by the curve OA . The asymptote is $y = \frac{2}{3}x + \frac{4}{9}$.

small ions with uncharged nuclei and with nuclei of unknown character respectively. It will be noticed that, when the combination coefficients are small,

$$y = 2x \text{ approximately.}$$

When the coefficients are large, $y = \frac{2}{3} (x + \frac{2}{3})$.

When the coefficients are small there is a superabundance of uncharged nuclei, but when the coefficients are large the numbers of uncharged nuclei and of large ions of either sign tend to equality. The limiting values of N/N_0 or $(1 + 3x)/(1 + x)$ are 1 and 3.

For comparison with observations we note that J. J. and P. J. Nolan found* in country air value of N/N_0 varying between 1.1 and 3, the mean ratio being 1.9. The value of N_0/N_1 corresponding with this mean is 2.2. The Nolans quote the following values found by other observers for N_0/N_1 , 2.4 (Gockel) and 2.2 (Hess), both in the open air, 1.3 (Nolan, Boylan and de Sachy) for the air of a closed room. Accepting the formula (7.2), we deduce that for country air, with $N_0/N_1 = 2.2$,

$$\eta_{10} = 0.83 \times 4\pi ew_1, \eta_{12} = 1.83 \times 4\pi ew_1, \beta = 0.87 \times 4\pi ew_1 N,$$

whilst for the air of a closed room in Dublin, with $N_0/N_1 = 1.3$,

$$\eta_{10} = 3.3 \times 4\pi ew_1, \eta_{12} = 4.3 \times 4\pi ew_1, \beta = 2.6 \times 4\pi ew_1 N.$$

These last results are equivalent to those deduced above on p. 372 from the same observations.

It is clear that the η 's and β are by no means invariable.

§ 8. EXCURSUS ON THE DETERMINATION OF THE DISSIPATION COEFFICIENT BY SCHWEIDLER'S SECOND METHOD.

Simultaneous observations of the dissipation coefficient and of the number of nuclei have been made by P. J. Nolan and C. O'Brolchain. In this research a method due to Schweidler and known as Schweidler's second method† was adopted. It is necessary to consider the theory of this method, for Schweidler's analysis appears to be wrong in one detail so that the rate of dissipation computed by his formula is twice as great as it should be. That the rate so computed is double that found by Schweidler's first method was demonstrated experimentally by O'Brolchain, but the origin of the discrepancy has apparently been overlooked.

The "second method" is quite simple in principle. One electrode of a closed condenser is connected to an electrometer, which is initially earthed, the other electrode is maintained at a fixed potential, and the current flowing into the former electrode is measured. Let v and C be the volume and capacity of the ionization chamber, V the potential difference and i the current. In accordance with our general notation q is the rate of production of small ions per unit volume, n_1 is the number of small positive ions per unit volume, whilst β_1 is the dissipation coefficient and w_1 the mobility. Let F be the strength of the electric field at a point on the surface S of the electrode, where the surface density of the charge is σ . Then $F = 4\pi\sigma$ and the total charge on the electrode is CV . Accordingly the current flowing into the electrode is such that

$$\frac{i}{e} = w_1 n_1 \int F dS = 4\pi w_1 n_1 \int \sigma dS = 4\pi w_1 n_1 CV \quad \dots\dots(8.1).$$

* *Proc. R. Irish Acad.* 40 A (1931) 33.

† *Wiener Berichte* 133, 23 (1924).

v, C
 V, i

F
 S, σ

Further, in the steady state all the small positive ions which are produced either are dissipated or flow to the negative electrode. Accordingly

$$qv = \beta n_1 v + i/e \quad \text{.....(8.2)}$$

From these equations it follows that

$$qve/i = 1 + \beta v/4\pi w_1 CV \quad \text{.....(8.3)},$$

or

$$i + Hi/V = qve \quad \text{.....(8.31)},$$

II where

$$H = \beta v/4\pi w_1 C \quad \text{.....(8.4)},$$

and H is independent of v .

In successive experiments V is varied and it is found that there is a linear relation between the values of i and i/V ; so H is readily determined and the value of β can be deduced from that of H .

In this argument it is assumed that the number of small ions is the same when averaged through the volume and when averaged through a thin layer in contact with the electrode. The justification for this assumption is that the air is more or less stirred up by convection currents, and the electric current carrying the ions towards the electrode is weak. Schweidler, on the other hand, ignored the diffusion of ions and regarded the current through the ionization vessel as carried by the transfer of ions of both signs under the influence of the electric field. Thus, according to his assumptions,

$$i = (w_1 n + w_2 n_2) e \cdot 4\pi CV.$$

Regarding n_1 and n_2 as equal, he gave a formula equivalent to

$$\beta = 4\pi (w_1 + w_2) CH/v$$

with $w_1 + w_2$ instead of our w_1 .

O'Brolchain's experiments* may be regarded as demonstrating that our way of regarding the matter is the right one. In these experiments O'Brolchain determined β by both of Schweidler's methods. In the "first method"† the ionization chamber is allowed to stand for a considerable time with no applied voltage and the number of small ions in the final state is found by sweeping them up, by applying a strong voltage for a short time. Denoting this number by $v(n_1)_0$ we have the equation

$$q = \beta (n_1)_0 \quad \text{.....(8.5)}.$$

In the second part of the experiment a considerable steady voltage is applied and the saturation current is determined. Writing $(i)_\infty$ for the strength of this current, we have the relation,

$$qve = (i)_\infty \quad \text{.....(8.51)}.$$

Accordingly

$$\beta = (i)_\infty / ve (n_1)_0 \quad \text{.....(8.6)}.$$

O'Brolchain found that the value of β determined in this way was about half of that derived from the formula

$$\beta = 4\pi (w_1 + w_2) CH/v.$$

Since w_1 and w_2 are nearly equal his observations serve to verify our formula (8.4)

$$\beta = 4\pi (w_1) CH/v.$$

* *Gerlands Beiträge*, 29, 1 (1931).

† *Wiener Berichte*, 127, 953 (1918).

In criticism of the analysis by which this formula was reached it may be said that the assumption of uniform distribution is inconsistent with the fact that the ions are being urged by the electric field towards one electrode or the other. Obviously the assumption is only an approximation to the truth, but as experiments demonstrate that the relation between i and i/v is linear it is likely that the approximation is a good one. The way in which the linear relation would break down under other assumptions may be illustrated by an example.

Let us suppose that the small ions are stratified but that the large ions are so nearly uniformly distributed that the variations of β are negligible. For simplicity let us take the case of a chamber in which the electrodes are parallel planes, so large that the edge effects may be ignored. Let the distance between the planes be h , and

h

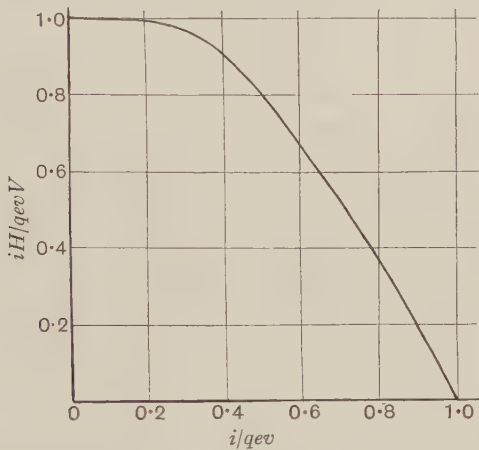


Figure 2. A hypothetical curve representing the relation between apparent conductivity and current, allowance being made for the stratification of small ions.

the area A , so that $v = hA$ and $C = A/4\pi h$. Let n_1 be the density of the positive ions at a distance z from the negative electrode. Let F be the electric force, which is uniform, the effects of the space charge being negligible. The density of the current carried by the positive ions is $n_1 ew_1 F$ and therefore

A
 z

$$q = \beta n_1 - \frac{\partial}{\partial z} (n_1 w_1 F) \tag{8.7}.$$

Subject to the condition that $n_1 = 0$ at the positive electrode ($z = h$), the solution of this equation is

$$n_1 = (q/\beta) [1 - e^{-\kappa(h-z)}],$$

where

$$\kappa = \beta/w_1 F.$$

κ

The value of n_1 close to the negative electrode ($z = 0$) is given by

$$n_1 = (q/\beta) [1 - e^{-\kappa h}].$$

Hence it follows that the current i is such that

$$i = (qevV/H) [1 - e^{-H/V}] \tag{8.8},$$

where

$$H = \beta v/4\pi Cw_1.$$

The relation between i and V is represented by the curve in figure 2 instead of by a straight line. Greater elaboration of the theory would probably lead to curves of the same type. The linear relation found by observation implies that the conditions are really simple. By that criterion our original assumption is justified.

For the purpose of the present paper it is convenient to regard observations by Schweidler's "second method" as giving the value of $\beta/4\pi w_1 e$ and the essential formula is

$$\frac{\beta}{4\pi w_1 e} = \frac{HC}{ve} \quad \dots\dots(8.9).$$

For the Dublin experiments C/ve was 7.18×10^5 in electrostatic units. The formula appropriate for use when H is expressed in volts is

$$\beta/4\pi w_1 e = 2.39 \times 10^3 H. \quad \dots\dots(8.91).$$

§9. VARIATIONS IN THE DISSIPATION COEFFICIENT FOR SMALL IONS

The Dublin Observations. Nolan and O'Brolchain give the results* of 31 observations of N and H in Dublin air. From these observations I have deduced the values of $\beta/4\pi w_1 e$ and of the ratio $\beta/4\pi w_1 e N$, denoted by y in § 7. The results are set out† in table 1. The extreme values of y are 0.27 and 1.32. The corresponding values of x being 0.175 and 1.48 (cf. figure 1), it is implied, if the theory is correct, that the ratio of the number of uncharged nuclei to the number of charged nuclei of either sign, $(1+x)/x$, varied between 6.7 and 1.7. The median value of y was 0.74, corresponding with $x = 0.68$ and $(1+x)/x = 2.5$. It will be noticed that this last estimate is nearly that given by Gockel, and quoted above, for the ratio N_0/N_1 .

Table 1. *The Dissipation Coefficient β . Results derived from observations made by "Schweidler's second method" by Nolan and O'Brolchain.*

N 10^3	H	$\beta/4\pi w_1 e$ 10^3	y or $\beta/4\pi w_1 e N$	N 10^3	H	$\beta/4\pi w_1 e$ 10^3	y	N 10^3	H	$\beta/4\pi w_1 e$ 10^3	y
53.4	20.0	47.8	0.90	25.7	10.0	23.9	0.93	21.5	4.0	9.6	0.45
40.0	4.5	10.7	0.27	25.7	7.0	16.7	0.65	21.2	11.7	28.0	1.32
36.5	12.3	29.4	0.80	25.0	6.0	16.5	0.66	21.2	9.0	21.5	1.01
36.3	13.5	32.2	0.89	24.5	6.2	14.8	0.60	21.2	8.2	19.6	0.92
35.5	8.4	20.0	0.56	23.8	7.3	17.4	0.73	20.0	6.0	16.5	0.79
34.0	9.1	21.8	0.64	23.6	5.5	13.1	0.56	20.0	9.4	22.5	1.08
34.0	6.0	14.3	0.42	23.6	6.8	16.3	0.69	20.0	7.7	18.4	0.92
32.2	6.1	14.6	0.45	22.0	9.2	22.0	1.00	17.8	5.5	13.1	0.74
29.8	7.9	18.0	0.63	22.0	9.4	22.5	1.02	17.5	6.2	14.8	0.85
29.0	6.0	14.3	0.49	22.0	10.0	23.0	1.09	17.0	6.1	14.6	0.86
29.0	8.5	20.3	0.70	—	—	—	—	—	—	—	—

The individual values of y are apparently not correlated in any way with the values of N . The assumption that the combination coefficients are invariable would evidently be far from the truth. It is to be expected that large ions with different histories will vary in size, and that the likelihood of collision between large and small ions, depending as it must on the size of these particles, will also vary.

* *Proc. R. Irish Acad.* 38 A, 40 (1929).

† The entries under N are to be multiplied by 1000 to give the number of nuclei per cm^3 .

With such an idea in mind P. J. Nolan kept air in the ionization chamber for several days and followed the changes in the number of nuclei and in the dissipation coefficient. The values of N and H which he obtained* have been copied in table 2.

It will be seen that in each series of observations the number of nuclei diminished. The coefficient y , which is proportional to the combination coefficient for small ions and nuclei, increased. As Nolan points out, this coefficient varies roughly as $N^{-\frac{1}{2}}$. The combination coefficient η_{10} for small ions and uncharged nuclei, which is proportional to the x of table 2, is found, with about the same accuracy, to vary as $N^{-\frac{2}{3}}$, the product $xN^{\frac{2}{3}}$, given in the last column of the table, being approximately constant in each experiment.

Table 2. P. J. Nolan's experiments illustrating the change of the dissipation coefficient in air kept in a closed vessel for two or three days.

Dates 1928	N $10^3 \times$	H	$\beta/4\pi ew_1$ $10^3 \times$	y or $\beta/4\pi ew_1 N$	x or $\eta_{10}/4\pi ew_1$	$xN^{\frac{2}{3}}$ $10^2 \times$
April 17-20	22.0	5.5	13.1	0.6	0.5	3.9
	2.94	2.0	4.8	1.6	1.9	3.9
	1.50	1.6	3.8	2.5	3.2	4.2
	1.13	1.05	2.5	2.2	2.7	2.9
Dec. 18-19	5.25	4.8	11.5	2.2	2.7	8.1
	3.27	3.7	8.8	2.7	3.4	7.5
	1.22	2.1	5.0	4.1	5.5	6.3
Dec. 19-20	18.7	7.2	17.2	0.9	0.9	6.3
	3.41	3.0	7.2	2.1	2.6	5.9
Dec. 20-21	3.47	2.4	5.7	1.6	1.9	4.4
	2.17	1.8	4.3	2.0	2.4	4.0
	1.12	1.25	3.0	2.7	3.4	3.7

If the nuclei in the ionization chamber diminished in number by combination, not by deposition on the walls of the chamber, then the masses of individual nuclei were, at any time, proportional to N^{-1} and the areas varied as $N^{-\frac{2}{3}}$. Accordingly the observations are consistent with the hypothesis that the combination coefficient for small ions and neutral nuclei is proportional to the area of the individual nucleus.

Thus the explanation of the variation in the ratio N_0/N_1 discussed in § 7 appears to be that the ions of the laboratory air were larger than those generally to be found in country air. The lesson of our table 1 is that, in Dublin at any rate, there is no correlation between the number of nuclei and their average size.

At Kew Observatory numerous simultaneous observations have been made of the conductivity of the air and of the number of Aitken nuclei. It has been disappointing to find that there is no simple law by which these data are associated. The recognition that nuclei with different origins and different histories are bound to have different combination coefficients should help us to explain the observations.

* *Proc. R. Irish Acad.* 38 A, 49 (1929).

§ 10. CONCLUSION

I have felt some diffidence in publishing the new formulae without waiting for further observations by which they can be tested. My excuse must be that although the needful observations are simple in principle there are likely to be practical difficulties which may not be overcome without considerable delay. I am therefore venturing to submit this paper for publication in the hope that other investigators, who may be provided already with suitable apparatus, will undertake experiments. Enough has been said to demonstrate that such experiments will help to fill up some of the large gaps in our knowledge of atmospheric electricity.

THE CRYSTALLINE STATE OF THIN SPLUTTERED FILMS OF PLATINUM

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ABSTRACT. Films of platinum spluttered in various gases have been examined by the method of electron-diffraction. They often show patterns which indicate that the small crystals are oriented with one face parallel to the surface of the specimen, the crystals being otherwise at random. The width of the rings formed by diffraction shows that in many cases the crystals are very small, of the order of 5×10^{-7} cm. Some films of platinum dioxide showed crystals of the order 2×10^{-7} cm.

§ 1. INTRODUCTION

IN the course of a research in collaboration with Prof. G. I. Finch and others on the catalytic properties of thin spluttered films of platinum, we have noticed certain peculiarities in the crystalline condition of some of the films which seem worthy of note. They concern the orientation of the small crystals of which these films are composed, and the size of the crystals.

The films were prepared by spluttering in a gas—argon, oxygen, nitrogen or hydrogen—at a pressure of about 0.1 to 0.5 mm. and at voltages varying from about 500 V to 3000 V in different cases. The crystalline nature of the films was investigated with an electron-diffraction camera which had previously been used by one of us to study the surface layers formed by chemical action, and also to investigate single crystals of metal*. Briefly the method consists in the use of the wave properties of the electrons to form diffraction patterns from the crystals in the surface layer of the substance to be examined, which, when that layer is polycrystalline, are strictly analogous to the Debye-Sherrer rings obtained with X-rays. With some of the specimens these rings were uniform and sharp, indicating that the crystals forming the surface layer were arranged at random, and that each contained a sufficient number of unit cells to make them act like gratings of high resolving power. In some cases, however, the rings were relatively broad. In other cases some at least of the rings showed marked variations in intensity along their circumference.

§ 2. WIDTH OF RINGS

Three principal causes contribute to the width of a diffraction ring: (1) The geometrical width of the electron beam as determined by the diaphragms which define it; (2) the inhomogeneity of the velocities of the electrons and consequent variation in wave-length; (3) the finite size of the diffracting crystals.

* G. P. T., *Proc. R. S.* 128, 649 (1930); 133, 1 (1931).

The angular breadth B due to the last cause is given by*

$$B = \frac{2\lambda}{D} \sqrt{\frac{\log_e 2}{\pi}},$$

D, λ
 N, a

where D is the thickness of each minute crystal assumed cubic in shape and λ is the wave-length. If we write $D = Na$, where a is the side of unit cube of the structure (3.91×10^{-8} for platinum),

$$N = 0.94 \lambda / aB.$$

The homogeneity of velocity depends on the constancy of the discharge; the latter was produced by a transformer feeding two condensers in parallel each of capacity $0.022 \mu\text{F}$, the frequency was $50 \sim$, and the reverse current was stopped by a valve. With a current of 0.3 mA , which was about that used, the variation in voltage is thus about 450. The broadening due to this at $30,000 \text{ V}$ for the rings usually measured is 0.07 mm. and is negligible. This conclusion is confirmed by the fact that the larger rings were as sharp as the inner ones, while the effect of inhomogeneity is proportional to the radius of the ring.

The extreme geometrical width of the electron beam was about 2.2 mm. , and the rings should be broader than this owing to diffraction occurring at different distances from the photographic plate on different parts of the specimen; actually most of the rings (see below) were less than 1.2 mm. wide. The beam was limited by two circular holes, so that it was strongly concentrated towards the centre; the width to half-intensity would be about 1.0 mm. , assuming uniform distribution of the electrons. It is well known however that such beams are often narrower than the geometry would suggest, owing to the curvature of the lines of force near the holes; and this must have been the case here, for rings were observed whose half-intensity width was as little as 0.6 mm. The width of the rings was measured with dividers, and since the edge was never sharp the best estimate was made of the half-intensity width by eye. Visual measurements have been shown to give good agreement with those made by means of a photometer, and in view of the large number of plates to be measured some quick method was necessary. The values found are shown in table 1.

Table 1.

Number of plates	Half-intensity width of rings (mm.)	Number of plates	Half-intensity width of rings (mm.)
3	0.7	13	1.2
5	0.8	3	1.3
12	0.9	1	1.4
39	1.0	2	1.5
9	1.1		

All these rings showed patterns undoubtedly due to ordinary platinum. It would have been rather difficult to make any certain deductions from these results in view of possible variations of the "geometric" width of the beam with the conditions of

* W. H. Bragg, *X-rays and Crystal Structure*, p. 133.

the discharge if it had not been for a fortunate accident. A number of the plates showed, in addition to the rings due to metallic platinum, other extra rings due to some substance not yet identified but probably a compound of platinum. These extra rings were always sharp even when the rings of normal platinum were not*. The measured widths were 0.8, 0.7, 0.8, 0.8, 0.6, 0.6, 0.6 mean 0.7 mm. The associated rings had widths from 0.8 to 1.2 mm. and in the case of one specimen 2.0 mm. and 2.2 mm. (on two plates). These last have not been included in the above table as the rings were so fuzzy that they could not be identified with complete certainty as due to ordinary platinum, though it is highly probable that they were. Since there is no reason to suppose that the extra rings, which depend on the specimen tested, should occur when the testing apparatus was working in any special way, we are entitled to take 0.7 ± 0.1 mm. as giving the width of the beam due to causes (1) and (2), and to attribute any extra width to cause (3). Thus we can say that the majority of the plates show broadening due to the small size of the crystals of the spluttered platinum.

To calculate the amount of this broadening we need to know the shape of the intensity curves due to the different causes. If both were error curves the combined breadth, b , would be given by $b^2 = b_1^2 + b_2^2$, where b_1, b_2 are the breadths due to the separate causes. If both were sharply defined the *total* breadth would be $(b_1 + b_2)$ and would be sharply defined. The actual beam is certainly not sharply defined at the edges. If all the crystals had the same size, the breadth due to this cause would have a fairly definite limit, but it is more likely that there is a graduation in sizes. It is probable that the truth lies between the two extreme assumptions.

b_1, b_2

Taking the first, a ring of width 1.0 would have $\sqrt{\{(1.0)^2 - (0.8)^2\}}$, or 0.6 mm. due to crystal size. This corresponds, when $\lambda = 0.6 \times 10^{-9}$ (which is an average value), to $N = 9.5$ unit cubes.

On the second assumption the same ring gives a width of $1.0 - 0.8$ or 0.2 mm., which gives $N = 28$.

For a width of 1.5 mm. the two methods give $N = 4.5$ and 8.1 respectively. Thus we may say that the majority of the rings were caused by crystals with about 20 unit cubes in the side, and a few by some with only about 6.

§ 3. RINGS OF PLATINUM DIOXIDE

In addition to the patterns which can be referred to platinum itself, others occurred when the spluttering was done in oxygen at a low or moderate voltage (less than 2000). Rings were obtained which there is reason to attribute to platinum dioxide. They were often quite sharp, especially when the platinum was spluttered below 900 V, averaging under 0.9 mm. At higher voltages they became very diffuse. Two rings were much stronger than any of the others, and these alone appeared distinct in the diffuse pattern, apart from some very faint rings farther out. The width varied from 1.5 to 2.6 mm., and the mean was 2.1. This corresponds to a size

* The following are the spacings corresponding to the main rings of this pattern (unit, 10^{-8} cm.): 4.4, 3.06 (s.), 2.06 (ms.), 1.73, 1.31 and many smaller.

of about 1.6×10^{-7} cm. for the small crystals. With the exception of one plate having rings 1.3 mm. wide there is nothing between these very diffuse rings and the sharp rings described above. This pattern was only found when the spluttering was in oxygen. On heating to 225° C. it passed into normal platinum, not orientated.

§ 4. ORIENTED FILMS

Variation in intensity round the circumference of the rings implies that the small crystals were not arranged at random. Examples of these are shown in figures 2 and 3. The extent of the orientation varies very much, from the case, such as that shown in figure 3, where the rings are almost reduced to isolated spots, to other more numerous cases in which there is a hardly perceptible difference of intensity along the circumference of some of the rings. The general nature of the orientation observable is in all cases the same, namely, that a single type of crystal plane is parallel to the free surface of the blank on which the platinum has been spluttered, the orientation being at random round the normal to this surface. In such a case the ring corresponding to the orientated plane will show a maximum of intensity in a direction normal to the surface. The rings corresponding to other planes will show maxima in different directions as can be seen from the figure, in which O , the centre



Figure 1.

of the rings, is the point in which the incident rays would meet the plate, AB is the intersection of the plane of the specimen, and the dotted lines represent the intersections of the Bragg planes with the plate. The intensity-maximum at C is due to planes which are (nearly) parallel to AB . The angle ψ between the radii corresponding to the directions of maximum intensity of different rings is equal to the angle between the Bragg planes corresponding to these rings. Thus we have

$$\cos \psi = \frac{j_1 j_2 + k_1 k_2 + l_1 l_2}{\sqrt{(j_1^2 + k_1^2 + l_1^2)} \sqrt{(j_2^2 + k_2^2 + l_2^2)'}}$$

where $(j_1 k_1 l_1)$, $(j_2 k_2 l_2)$ are the Miller indices of the Bragg planes causing the rings at C and D . Although the general nature of the orientation is always the same, the preferred plane differs in different cases. In the course of the investigation of catalytic activity 93 specimens in all were tested; of these 32 showed appreciable orientation (see table 2).

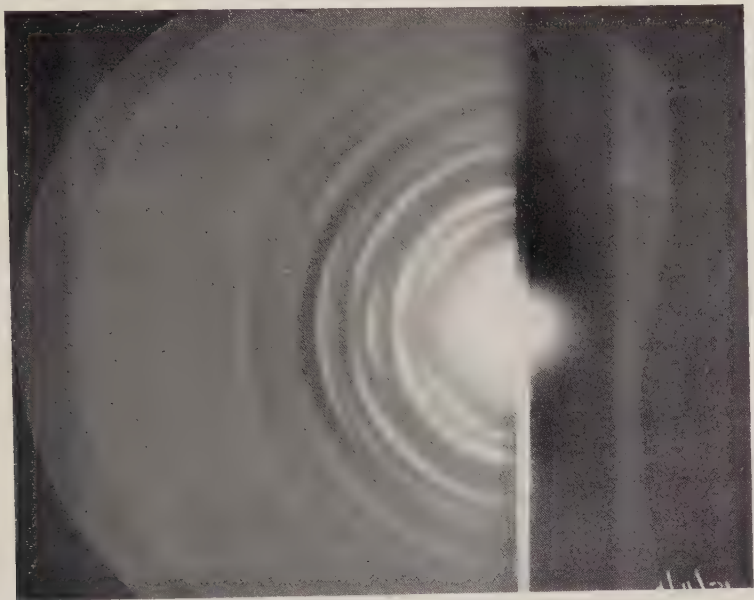


Figure 2. (100), Cubic face parallel to face of specimen.

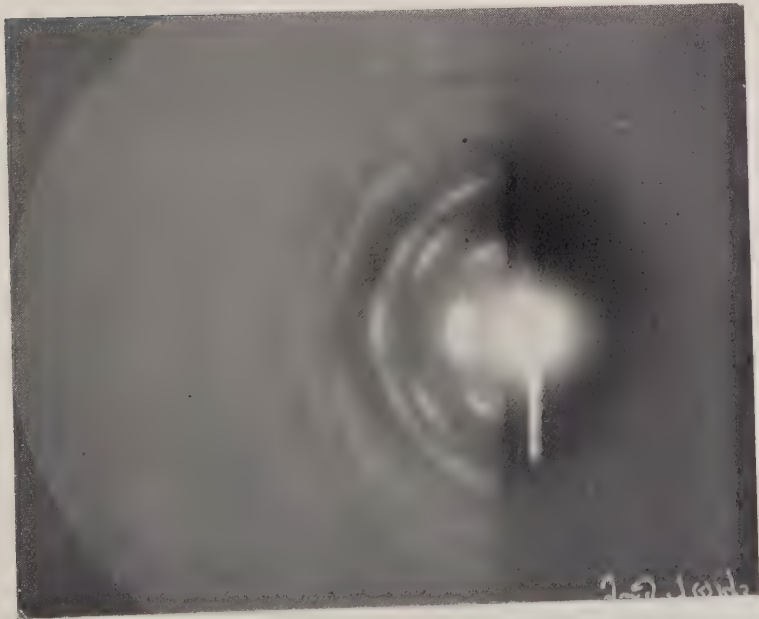


Figure 3. (111), Octahedral face parallel to face of specimen.

Tables 3 and 4 give typical measurements from some of the specimens and show agreement between the calculated and observed directions of the maxima on the various rings. The error in determining the position of the maxima depends greatly upon the extent to which the intensity is concentrated, but is seldom less than 2° or 3° , so that the agreement is as good as can be expected.

Table 2.

Number of plates	Number of specimens		Face parallel to blank surface
16	4 7 2	On quartz On glass On molybdenum	(100), cube
11	3 5 3	On quartz On glass On molybdenum	(111), octahedral
5	3 1	On glass On molybdenum	(110)
5	2	On glass	(100) and (420)
5	2	1 on glass 1 on quartz	{ (111) and (110) with (100) and (331) in one case

Table 3. *Plate of November 9, 1931. Cube face parallel to surface.*

(hjk)	$\pm \phi$ observed	$\pm \phi$ calculated
(111)	C. 60° (too black to measure exactly)	54.7°
(200)	0°	0°
(220)	45°	45°
(311)	$25^\circ, 70^\circ$	$25.2^\circ, 72.5^\circ$
(222)	60°	54.7°
(400)	0°	0°
(331)	C. 50°	$46.5^\circ, 76.7^\circ$
(420)	C. 30°	$26.6^\circ, 63.4^\circ$
(511)	C. 20°	$15.8^\circ, 78.9^\circ$

Table 4. *Plate of July 19, 1932. Octahedral face parallel to surface.*

(hjk)	$\pm \phi$ observed (mean)	$\pm \phi$ calculated
(111)	$0^\circ, 68^\circ$	$0^\circ, 70.5^\circ$
(200)	52°	54.7°
(220)	37°	35.3°
(311)	$30^\circ, 60^\circ$	$29.5^\circ, 58.5^\circ$
(222)	0°	0°
(331)	22°	$22.0^\circ, 48.5^\circ$
(420)	40°	39.2°

§ 5. DISCUSSION OF RESULTS

It is probable that the orientation, when it occurs, is a thermal effect produced during or after the deposition of the platinum. As the main object of the research was an investigation of the catalytic properties of the films, with which their orientation shows no connection, no special attempt was made to investigate the temperature conditions, but it was noticeable that the films which showed orientation were usually those which had been prepared at high voltages and current, and for which the cathode had been visibly hot. It is probable that the orientation occurs best over a definite range of temperature. Strong orientations occurred in two or three cases in which the cathode was at a white heat, but in one case when the cathode was dazzlingly bright no orientation was observed.

The blanks used were of three materials, molybdenum, quartz and glass. The quartz and glass blanks were cleaned in fresh aqua regia for some time before use, rinsed with distilled water, and dried on clean filter paper. The molybdenum was polished with Bluebell polish rinsed in alcohol, and rubbed on cotton wool. There was no correlation observable between the nature of the blank and the orientation, either in the frequency in which the orientation occurred or in the preferred plane. This is shown in table 2. It is doubtful if the nature of the gas has any marked effect on the types of orientation, all of which have been found with argon, and though the oxygen films show only the octahedral type and the nitrogen only the cubic type it is doubtful if much weight should be assigned to these differences, as voltage and pressure conditions were such that comparatively few of the films showed orientation at all. When several photographs were taken from one specimen the orientation shown was always the same, so that the state of the film is fairly uniform over the surface of the specimen, which was usually about 1 cm. × 1 cm. It should be emphasized that what we have here is not the formation of a film consisting of a single crystal as found in the early experiments of one of the authors* and investigated recently by Trillat and Hirsch†. Our films are analogous to those studied by Kirchner‡, who however does not appear to have found more than one kind of orientation for each of the substances which he tested.

The diffraction patterns from a single crystal, as one of us has shown§, are of quite a different character. A very interesting question, and one which the present research does not answer, is—what determines the face which takes up the preferred position parallel to the surface of the blank? If it were a question of pure chance we should expect that, since the cubic and octahedral faces are both common, the usual type of pattern would be a mixture of the two, but in fact this is very rare if it ever occurs. It appears as if some feature in the conditions determines the face. The most obvious is the temperature, but for the reason stated we have no evidence on this point. It would be a matter of great interest to investigate further in this

* G. P. T., *Proc. R. S.* **117**, 600 (1928).

† *J. de Phys.* **111**, 1932, p. 185.

‡ *Z. f. Phys.* **76**, 576 (1932).

§ G. P. T., *Proc. R. S.* **133**, 1 (1931).

direction, and the present results are offered merely as an indication of what may be expected. Their value lies in the number of specimens which have been tested and the wide range of conditions, which have resulted in a variety of patterns being found. They represent a by-product from another research and are in no sense exhaustive.

The relation of the ring pattern to the catalytic activity of the film, which was the main object of the research, will be discussed in a forthcoming paper with Prof. G. I. Finch and others, but it may be remarked here that the size of the crystals appears to have no connection with the activity, which depends on the nature of the film.

§ 6. ACKNOWLEDGMENTS

In conclusion, one of us (N. S.) would like to take this opportunity of thanking the Salters' Institute of Industrial Chemistry for the award of a Fellowship, during the tenure of which part of this research was pursued. Our thanks are also due to Mr T. Riches for his assistance in the manipulation of the apparatus.

DISCUSSION

Prof. E. V. APPLETON. I wondered, on looking at the diagram of the apparatus, why a cold-cathode discharge was used in preference to a thermionic source with high vacuum conditions. The presence of gas may make it easier to get a well defined electron beam, as in modern cathode-ray oscillographs, but is it not possible that the presence of gas at the same time affected the surface of the specimen?

Dr M. C. JOHNSON. As the size of the particles in the surface films proved to be so small, whereas cathode-sputtered material has previously been expected to be of larger colloidal dimensions, would the authors give any detail of the method of deposition adopted to secure such small crystals? It is known that films deposited from a stream of separated atoms of metallic vapour can consist of minute crystals, which grow by the surface migration of adsorbed atoms. Have the crystals described in this paper grown similarly, or is their final size determined by their size as multi-atomic aggregates before striking the surface? If the cathode particles are as large as they have generally been supposed to be, and yet form deposits whose individual particles are smaller than those which grow after being deposited from purely atomic streams, the structure of the surface film seems independent of all history before impact and may depend on the temperature and smoothness of the target above.

Prof. G. I. FINCH. As regards § 3, in the course of recent experiments Mr A. G. Quarrell and I have found that the definition of the diffraction rings due to cathodically sputtered platinum dioxide, and hence the crystal-size, depends in the main upon, and decreases with, the rate of deposition. We have observed a similar effect with both sputtered and evaporated metal films, provided the receiver was cool.

Heating the receiver usually brought about a growth of the crystal-size in the deposited film and frequently resulted in orientation. Thus it does not seem that the crystal-size can be properly ascribed to the discharge characteristics, except in so far as these may afford a measure of the rate of deposition. At the end of § 5 the view is expressed that the size of the crystals appears to have no connection with the (catalytic) activity. Messrs Coles, Quarrell and I have, however, recently carried out further experiments, the results of which strongly suggest that crystal-size and activity are in fact intimately associated with each other. In collaboration with Dr Hodge* and Dr Thompson† I have shown that metals are sputtered as atoms.

With regard to Prof. Appleton's question concerning the cold cathode, Mr Quarrell and I prefer this type of discharge tube for the range of accelerating voltages we require (15–80 kV.) on account of its simplicity and the avoidance of light-fogging of the photographic plate. In conjunction with the cold cathode we also employ a diode which acts as a current-limiting device. It will be readily seen that the combination of a cold-cathode discharge tube supplied through a saturated diode combines the advantages of both the cold and hot-cathode types of discharge tubes without their disadvantages, and renders simple the attainment of a perfectly steady accelerating voltage.

AUTHORS' reply. In reply to Prof. Appleton: The gas-filled tube is on the whole easier to control than the hot-filament tube. We have never observed any effect of gas on the specimen, possibly because the impact of the cathode rays cleans the surface, though effects have of course been observed with slow electrons.

In reply to Dr M. C. Johnson: The experimental conditions of deposition were not intentionally designed to produce films with any particular crystal-size. Sputtering was carried out with a platinum wire cathode of length 5.5 cm. and diameter about 0.75 mm. The films were usually deposited on to either glass or quartz-plate receivers, optically flat and polished. Molybdenum and silver blocks, also polished, were used at one time and gave the same results. Films sputtered in oxygen at the lowest possible discharge currents and potentials were found to give very sharp diffraction patterns of oxide. At high currents of the order of 10 mA. and potential of the order of 1800 V., very diffuse patterns were obtained. These results might be explained by analogy with the connection between crystal-size and rate of growth in solution. In this case the rate of sputtering would be the controlling factor.

As Prof. Finch has said, it is now generally accepted that sputtering takes place by individual atoms, so there is nothing surprising in the final aggregates being small. Probably heat treatment, when it occurs, makes them grow larger. As regards the influence of particle-size on activity, the statement was only meant to apply to the range of size considered in this paper, and more especially to the platinum particles.

* *Proc. R. S., A.* **124**, 303 (1929).

† *Proc. R. S., A.* **129**, 315 (1930).

WEEKLY MEASUREMENTS OF UPPER-ATMOSPHERIC IONIZATION

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ABSTRACT. A series of weekly measurements of the maximum ionization content of the Kennelly-Heaviside region of the ionosphere is reported and discussed. The ionization is found to be 2.2 times as intense on a summer noon as on a winter noon, and, in general, slightly less in 1932 than in 1931. This reduction is considered to be probably due to the approach of sunspot minimum and, together with other evidence, suggests that the ionizing agency from the sun varies by as much as 60 per cent during the 11-year solar period. Although ultra-violet light is accepted as the major ionizing agency, thunderstorms most probably constitute one of the subsidiary causes. The possibility that thunderstorms might cause upper-atmospheric ionization has been previously suggested by C. T. R. Wilson.

§ 1. INTRODUCTION

IN a recent paper* we have described an experimental method of estimating the maximum ionic content of the Kennelly-Heaviside region of the ionosphere. The possibility of making such a determination arises as follows. If we project vertically upwards wireless waves of gradually increasing frequency (and therefore gradually shorter wave-length) we find that for a certain critical frequency penetration of the Kennelly-Heaviside region takes place, and reflection begins from an upper region in which the ionization is usually more intense. Such penetration is therefore indicated by a discontinuity in the curve in which equivalent height is plotted as a function of electric wave frequency. From this curve the critical frequency at which the penetration of the lower region begins is found by inspection, and from this value the maximum ionization content of the region is deduced.

§ 2. THE RELATION BETWEEN MAXIMUM IONIZATION AND CRITICAL PENETRATION FREQUENCY

The way in which the maximum ionization is estimated when the critical penetration frequency is known has been previously discussed at length for the case in which the ionization consists of electrons and also for the case when it consists of ions of molecular mass, so that only a word or two of explanation is required here. Although the problem cannot as yet be fully dealt with in terms of a wave treatment, as opposed to a simple ray treatment, we assume that reflection at vertical incidence takes place at the level at which the refractive index is equal to zero. Thus if we find the frequency which just penetrates, say, the lower region, we assume that for

* *Proc. R. S., A.* **137**, 36 (1932).

frequencies just less than this the refractive index is reduced to zero at the level of maximum ionization. For this condition we have, as has previously been shown

$$N_{\max} = \frac{3\pi m}{2e^2} f^2,$$

where N_{\max} is the ionic (or electronic) content per cm^3 , e and m are the charge and mass respectively of an ion (or electron), and f is the critical penetration frequency. The relation between N_{\max} and f is shown in figure 1, the broken curve referring to ions (oxygen atoms) and the continuous curve to electrons.

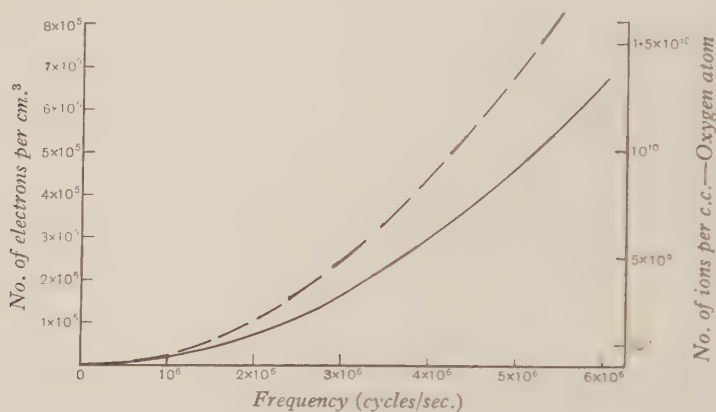


Figure 1.

In the present paper we are not concerned with the absolute values of N_{\max} but only with its variation.

§3. WEEKLY MEASUREMENTS OF MAXIMUM IONIZATION

In our previous work the measurements of the maximum ionization in the Kennelly-Heaviside layer were made over the whole 24-hours for selected winter, spring and summer days. These measurements showed that the ionization is a maximum about noon and falls off steadily as sunset approaches. The ionization continues to fall rapidly after sunset until an almost stationary night value is reached. Before and during ground sunrise the value of N_{\max} increases rapidly, and after sunrise more slowly, until the maximum noon value is again reached. We have pointed out that the general diurnal variation curves with their marked correlation with sunset and sunrise leave little doubt that the chief agency responsible for the ionization is of solar origin and travels through the earth's atmosphere in straight lines. We have also given reasons, based on the observations made by one of us during the solar eclipse of 1927, for concluding that this agency is ultra-violet light. That this view is substantially correct has been shown by the observations made by Dr J. Henderson in Canada, and by others, during the eclipse of the sun on August 31, 1932.

But in discussing our measurements of diurnal variation we pointed out that although there was a seasonal variation, the ionization at summer noon being about $2\frac{1}{2}$ times that at winter noon, we had encountered many days when the ionization appeared to be in excess of the normal seasonal value. We also recorded that on a number of nights the ionization in the lower region actually may increase at a time when any solar radiation propagated rectilinearly could not possibly be reaching the area immediately over the place of measurement.

In order to investigate the possible subsidiary ionizing agencies for *E* region we have made weekly estimates of the ionization throughout a period of nearly two years, from January 1931 to November 1932. These have all been made at the Radio Research Station, Slough, with signals from the emitting station at the National Physical Laboratory. The measurements were made every week on either Thursday or Friday round about noon.

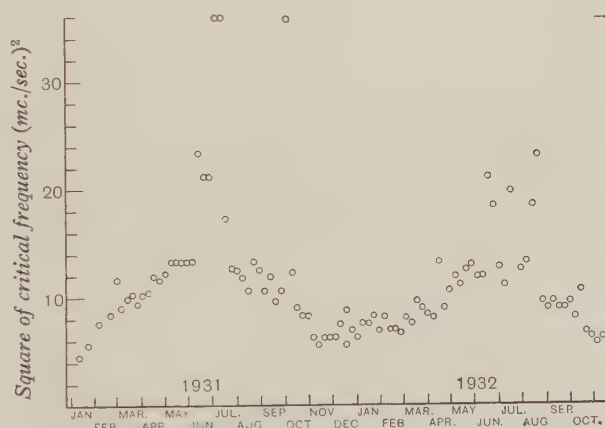


Figure 2.

In figure 2 the variation of the square of the critical frequency (which is proportional to maximum ionization content and may be translated into such values with the aid of figure 1) is shown over the whole period of observation. It will be noted that the seasonal variation is clearly marked, the ionization being more intense on a summer noon than on a winter noon. In our previous work on this subject we were led, from a comparison of the average results obtained on summer and winter days, to conclude that the summer-noon ionization was of the order of 2.9 times that on a winter noon. The more extensive data now available and shown in figure 2, however, show that abnormally high values of ionization are very frequently encountered in summer, so that if we wish to consider the undisturbed seasonal variation it is preferable not to take average values but to assess it from the general trend of the curve*. If we do this we find a somewhat lower figure for the ratio of summer to winter ionization, of the order of 2.2.

* We shall discuss more fully our reasons for doing this later in the paper, when we consider the possible nature of the sources of this abnormally high ionization.

If we compare the results obtained in 1931 with those obtained in 1932 we find that the general level of ionization is lower in the latter series. This is most easily seen by a comparison of the spring and autumn values. We have reasons, which we shall explain later, for excluding the summer values from such a comparison. It is not possible to give an exact figure for the average reduction from 1931 to 1932, but the diminution is probably of the order of 10 to 15 per cent. It is natural to associate this with the general fall in solar activity in the sequence of the solar cycle, since the last sunspot maximum was in 1928 and the succeeding sunspot minimum is expected in 1933-4.

If work in future years confirms this variation with the sunspot cycle, and we are able to ascribe it even only in part to the ultra-violet light from the sun, it will follow that such radiation varies with solar activity over a wider range than does the so-called solar constant.

In this connexion it is interesting to recall that L. W. Austin and I. G. Wymore* have shown that the signals from Nauyen, received at Washington, are approximately 1.8 times as intense at sunspot maximum as at sunspot minimum. Since waves of 13,000 metres are used in this case we can safely assume that the Kennelly-Heaviside layer is the reflecting region in question. Using G. N. Watson's theory of the propagation of waves between two reflecting spherical surfaces, one of the authors† has shown that the result obtained by Austin and Wymore may be interpreted as indicating that the conductivity of the lower reflecting layer is 1.6 times as great at sunspot maximum as at sunspot minimum. Such a variation of ionization is clearly not incompatible with the change we have noted between the ionization in 1931 and that in 1932‡.

In such terrestrial measurements of the intensity of the sun's ultra-violet light as have been published we do not, at present, find any support for the view that such radiations vary with sunspot activity. G. Pettit§ has measured the ratio of the ultra-violet radiation transmitted by silver ($\lambda 0.32\mu$) to the green light transmitted by gold ($\lambda 0.5\mu$) on many days since 1924, and although the early observations seemed to indicate that this ratio varies with solar activity it is now considered that the variations observed are more probably due to changes in the atmosphere. Further work is necessary before it is possible to assess from ground measurements how much of the variation is due to terrestrial causes, such as changes in the amount of ozone, and how much is really due to changes in solar emission.

It is also of interest to compare the above measurements of critical frequency which were made in South-East England with those which we have deduced from the series of measurements of equivalent heights at different frequencies made at

* *Proc. Inst. Rad. Eng.* **16**, 166 (1928).

† E. V. A. *International Research Council Second Report on Solar and Terrestrial Relationships*, p. 16 (1929).

‡ Although we are inclined to interpret our results as indicating quite a substantial variation in the sun's ultra-violet light during the solar period, it should be mentioned that the fact that the variation of solar control over radio transmission during this period increases with latitude suggests that charged particles must also be taken into account.

§ *International Research Council Third Report on Solar and Terrestrial Relationships*, p. 105 (1931).

the Bureau of Standards, Washington, and communicated to us by Science Service, U.S.A.

Unfortunately, measurements in Washington are made not at local noon but at a time which is two to three hours after mid-day, so that the comparison has to be made with caution. We have, however, compared our measurements of noon ionization with the American measurements of afternoon ionization, and find them to be of the same order of magnitude when corresponding periods are compared, so that we may conclude that noon ionization in Washington is on the average greater than noon ionization in England. This result is readily explained if we assume that the ionizing agency is ultra-violet light, and remember that Washington is situated in latitude $38^{\circ}8'$ N whereas our measurements were made in latitude $51^{\circ}5'$ N.

§4. THE CAUSES OF ABNORMAL IONIZATION

Although it is clear that the major source of daytime ionization for the lower region is ultra-violet light, evidence has been steadily accumulating which indicates that it is necessary to take other possible causes into account to explain departures of the ionization from the normal seasonal and diurnal values. Much of this evidence has accrued from observations made at night when, the level of normal ionization being low, departures from it are readily recognized. For example, in 1930 it was pointed out by one of us*, as a result of a long series of measurements of the equivalent height of *E* region, that on some occasions "either the recombination of ions is prevented or there is some ionizing agent present which can influence the dark side of the earth." The same matter was mentioned in our paper referred to above where we conclude that "there is some agency which produces nocturnal ionization beyond the ordinary amount constituting the residue from the day-time." Schafer and Goodall† and Ranzi‡ have also noted the same effects and arrived at similar conclusions. Notable interest, however, is to be attached to the work of Ranzi, since he has suggested that this source of nocturnal ionization is possibly connected with the troposphere. From a series of observations on 330 occasions Ranzi concludes that the abnormal night ionization in region *E* is associated with the presence of barometric depressions either at the point of observation or at a place to the north of it. This result is of special importance in connexion with the reasons advanced, as far back as 1924, by C. T. R. Wilson for concluding that thunderclouds contribute ionization to the atmospheric conducting layer, in the form of either intense ionization currents or lightning flashes. It is therefore of interest to test a fairly substantial set of data to see whether there is evidence that thunderstorms constitute the essential link in the correlation between barometric depressions and abnormal ionization. Our data for the period January 1931 to November 1932 were handed to Mr F. E. Lutkin for general statistical examination. He called our attention to a note which he had communicated to the Superintendent of the Radio Research Station on September 2, 1932. In this note he had reported

* E. V. A., *Proc. R. S.*, A. **126**, 567 (1930).

† *Proc. Inst. Rad. Eng.* **20**, 1131 (1932).

‡ Ranzi, *Nature*, **130**, 368 (1932).

abnormal shortening of the skipped distance for 15-mc. sec. waves which had been observed by a British wireless amateur, Mr E. R. Peach, in the thunderstorm conditions of August 18-19, 1932, and had suggested that this might indicate specially intense ionization, in the ionosphere, associated with local thunderstorms. It will be seen from figure 2 that the critical frequency on August 18 was in fact very high. In a further note dated September 12, Mr Lutkin had remarked on the high value of critical frequency, the low attenuation of 15-mc. sec. signals, and the high thunderstorm activity which had marked the preceding day, September 11.

In order to test these points further, he then compared our data with thunderstorm reports, and found such a close relation between days of high critical frequency and days of British thunderstorms that he undertook a detailed comparison of the data for the period January 1 to July 31, 1932, with a "thunderstorm index figure" derived as follows. During this period observations on the distribution of European sources of atmospherics were made at Slough for half-an-hour near 13h G.m.t. each week day. The peak amplitudes of all atmospherics recorded in this operation as coming from sources within 3000 km. were summed to form the thunderstorm index figure. The relation between ionization density in *E* region and this thunderstorm index was found to be represented by a correlation coefficient so high as 0.75. The comparison was not carried further because of the interruption of the direction-finding work after July and the relative inadequacy of sampling before January. It is, however, noteworthy that the occasions of abnormal ionization of *E* region in thunderstorm conditions on the dates in August and September already mentioned and a similar occasion on October 11 are not included in the period for which the high correlation coefficient was obtained.

In view of the above conclusion we are now inclined to attach special importance to a series of signal-intensity measurements made by Dr J. Hollingworth and one of us on signals from the station of St Assise ($\lambda = 14,350$ m.) received at Slough. These observations were such that it was possible to deduce the values of both ground wave and down-coming wave. One afternoon in May 1929 a thunderstorm area passed between the two stations when observations were in progress, and it was noticed that when the area reached the great circle joining the stations, atmospheric absorption became so pronounced that the reflected wave was undetectable. Although this is only an isolated case we believe it to be evidence of intense ionization at a considerable height, since the absorptive and refractive influence of ionization only become of importance when the pressure is so reduced that the time between two collisions of an ion with gas molecules is of the order of the electric wave frequency.

As we have already recorded, we have frequently found that a magnetic storm has been accompanied by abnormally high ionization in the lower region, though a correlation is by no means invariably found. We can only say that if there is a very big magnetic storm the ionization in region *E* is increased. In this connexion it is important to note that when from any cause, such as a magnetic storm or thunderstorm, *E*-region ionization is increased, there is not usually a corresponding increase in region *F*. In fact we have on some days found conditions the reverse of normal,

E -region ionization being greater than that in F region. We can therefore safely conclude that the agency or agencies which cause such abnormal ionization in region E is not the same as the agency (i.e. ultra-violet light) which ionizes both E and F regions in the normal manner.

The known influence of solar activity in terrestrial magnetism, which is shown by recurrence tendencies connected with the solar rotation-period of 27 days, naturally suggests the examination of our maximum ionization data for such recurrence tendencies. It has already been shown by various writers that periodogram analysis of long-distance signal-intensity data show evidence of such a periodicity corresponding to solar rotation. In this connexion it should, however, be pointed out that signal intensity is often an ambiguous index of ionospheric ionization, for it is well known that increase of ionization may, owing to absorption limitation, result in a reduction of signal intensity, whereas in other cases such an increase, by the removal of electron-limitation, may cause just the reverse result. No such objection applies to ionospheric-ionization measurements for the lower region as deduced from the critical penetration frequency, so that such data are of the type most suitable for comparison with other geophysical data. But although an examination of the smooth curve drawn through the weekly values does give a slight suggestion of a recurrence tendency with a period of 27 days, we do not believe that the data are sufficiently detailed for this result to be accepted with confidence. It is quite clear that daily readings are necessary, and it is hoped that it will be possible to obtain them in the future.

§ 5. ACKNOWLEDGMENTS

The work described in the paper was carried out as part of the programme of the Radio Research Board of the Department of Scientific and Industrial Research. We wish to express our indebtedness to Mr R. A. Watson Watt, Superintendent of the Radio Research Station, Slough, for providing us with facilities for the measurements, for making available the data for the "thunderstorm index figure," and also for helpful discussion at various stages of the work. We also wish to thank Dr R. L. Smith-Rose for arranging for the special use of the National Physical Laboratory emitter in connexion with these experiments.

Our special debt to Mr F. E. Lutkin, of the Radio Research Station, Slough, has been mentioned in the text, while to Mr D. Brunt, of the Meteorological Office, we owe thanks for assistance in the discussion of the results.

APPENDIX

In a paper previously referred to* C. T. R. Wilson discussed the effects of thunderclouds on the rarefied regions of the upper atmosphere. Even if there were no ionized regions due to other causes, Wilson showed that the electric forces due to thunderclouds were sufficient to cause one. Also, he showed that if an ionized

* *Discussion on ionization in the atmosphere*, Nov. 28, 1924. See *Proc. Phys. Soc.* **37**, 32 D. (1925).

region already existed the electric forces between such a layer and the thundercloud would produce ionization by collision, so that discharges between the cloud and the layer would probably take place.

It is of interest to consider in greater detail the processes pictured by Wilson, taking into account the variation of air pressure between the cloud and the layer. In doing so we shall be obliged, to some extent, to idealize the problem. We shall assume that the layer consists of highly-ionized air and plays the part of the filament in a large-scale diode, the positive head of the thundercloud acting as the corresponding anode. The actual state of affairs is shown inverted in the accompanying figure.



Figure 3.

i, h, σ_0
 σ
 H, E, ρ
 k
 k_0

Let i be the current per unit area flowing in the direction of the height h . If σ_0 is the pressure of the air at the level of the layer, the pressure σ at height h is equal to $\sigma_0 e^{h/H}$, where H is of the order of 10 km. Let E be the electric force, ρ the space-charge density, and k the ionic mobility at any height. If we assume that the mobility is proportional to the pressure, k is equal to $k_0 e^{h/H}$, where k_0 is the mobility of the ions at the level of the ionized layer.

From Poisson's equation we have

$$dE/dh = 4\pi\rho \quad \dots\dots(1).$$

Also the equation of current continuity may be written

$$i = k_0 e^{-h/H} \rho E \quad \dots\dots(2),$$

so that

$$\frac{dE}{dh} = \frac{4\pi i e^{h/H}}{k_0 E} \quad \dots\dots(3),$$

or

$$E^2 - E_0^2 = \frac{8\pi i H}{k_0} (e^{h/H} - 1) \quad \dots\dots(4),$$

E_0 where E_0 is the field at the surface of the layer. If the current is space-charge limited, E_0^2 is equal to zero, so that we have

$$E^2 = \frac{8\pi i H}{k_0} (e^{h/H} - 1) \quad \dots\dots(5),$$

as the representative space-charge equation.

To make an estimate of the vertical current we assume that the field near the thundercloud is, say, 1/10 of the sparking value, that is 10 e.s.u. cm.; we take H as 10 km. and h as 80 km. The value of k_0 is most uncertain, but if the pressure of the air at 80 km. is of the order of 0.02 mm., k_0 must be at least 10^7 e.s.u. Thus i calculated from (5) is 1.3×10^{-2} e.s.u. or 4.3×10^{-12} A./cm.²

If the field increases two-fold this current is increased four times. But on the other hand the estimate is probably a high one, in view of the fact that we have considered the positive charge on the cloud to act as an infinite plane. According to Wilson's theory of the maintenance of the earth's electric charge and the air-earth current we are to regard thunderstorms as supplying positive ions to the upper layer, where they diffuse all over the globe and become the downward current in regions where there are no thunderstorms. We can, however, easily see how a thundercloud can contribute the $\frac{1}{2}$ A. of current which the 2000 thunderstorms, in action at once, are required to supply in order to maintain the whole downward air-earth current of 1000 A.

It should be noted that either in the thunderstorm case or in the fine-weather case there are to be found space charges both near the ground and near the layer. It is easily shown that there is a certain height at which the space charge is a minimum. From (2) and (5) we have, eliminating E ,

$$\rho = \sqrt{\left(\frac{i}{8\pi k_0 H}\right) \cdot \frac{e^{h/H}}{\sqrt{(e^{h/H} - 1)}}} \dots\dots(6),$$

which expression is a minimum when $e^{h/H} = 2$, that is to say about 7 km. below the layer boundary.

It is also of interest to follow up Wilson's suggestion that the ionization current between the layer and the upper part of the cloud will be augmented by ionization by collision. This should most readily take place where the ratio of the electric force to the pressure is a maximum, that is to say, where $E/\sigma_0 e^{h/H}$ is a maximum. With the aid of (5) it is easily shown that this condition obtains at $h = 0.7 H$, or at a distance of 7 km. below the layer.

Let us compare the values of E/σ at this height of 7 km. and also at the top of the thundercloud where $h = 80$ km.

We have, on substitution,

$$\left(\frac{E}{\sigma}\right)_{7 \text{ km.}} / \left(\frac{E}{\sigma}\right)_{80 \text{ km.}} = 27.$$

In other words, when the field near the cloud is only $1/27$ of the sparking value, the field at a distance of 7 km. from the layer has reached sparking value and the production of intense ionization currents takes place. This result suggests that perhaps the first effect of a thunderstorm would be to produce a subsidiary region of ionization about 7 km. or so below the Kennelly-Heaviside layer.

The electric charge required to give the increased ionization observed is considerable. The volume in question cannot be much less than 1 km^3 and a simple calculation from the wireless data shows that the electric density must be of the order of 100 coulombs (of electrons) per km^3 or 10^6 coulombs (of ions) per km^3 . The simple discharge of the positive head of a thundercloud into the upper layer is evidently not sufficient to produce such intense local ionization, and it is clear that the electric charges must be considerably augmented in some way by the processes of ionization by collision.

DISCUSSION

Prof. S. CHAPMAN. I am much interested in the authors' estimate of the variation of ionization in the *E* layer in the course of the sunspot-cycle; it is of the same order as the sunspot-cycle variation in amplitude of the solar diurnal variation of terrestrial magnetism. The corresponding lunar diurnal magnetic variation, on the other hand, shows a much smaller dependence on the sunspot epoch, and for long I have interpreted this as indicating that there are different ionizing agents in the two regions in which these magnetic variations arise. I hope that radio evidence bearing on this difference will in time become available.

Dr D. OWEN. It appears from the formula on which the disseminations of maximum ionization content are based that an estimate should be attainable of the thickness of the lower or *E* layer of the ionosphere. According to the interpretation given on the first page of the paper, as the critical frequency of the electric waves rises, so the reflecting stratum concerned gets higher. If, then, the time of the double journey is measured for a series of frequencies up to that required for penetration of the layer, the depth of this should be deducible. It would be interesting to know if the authors could give an estimate of the thickness obtained in this way, and of its dependence on time of day or the seasons of the year.

Mr F. E. LUTKIN. In connection with the seasonal variation in ionization of the *E* region as shown in figure 2, while at first sight the annual variation immediately drives one to attribute this to changes in the sun's zenith distance, in view of the close correlation, mentioned in the text, between the thunderstorm index figure and ionization for each day irrespective of abnormality, I would suggest that some at least of the seasonal variation may be attributed to the seasonal variation of thunderstorm activity. The fact that the mean summer (July-August) level of the thunderstorm index amounts to 3.1 of the winter (November-December) value is in itself significant. The higher ionization at Washington, referred to in the paper, might also be looked for in view of its proximity to the West Indian tropical thunderstorm belt.

AUTHORS' reply. The agreement between our estimate of the variation of ionization in region *E* over the sunspot cycle with that previously calculated by Professor Chapman for the corresponding variation in conductivity of the layer responsible for the solar diurnal magnetic variation is perhaps one further example of the gratifying way in which the results obtained by magnetic and wireless methods are converging. Curves in which equivalent height is plotted against electric wave frequency can be drawn, so that it is possible to estimate the depth of penetration as suggested by Dr Owen* but as yet no data are available which would enable one to decide whether this depth varies diurnally and seasonally. We certainly agree with Mr Lutkin that it is not improbable that thunderstorms may be contributing to the seasonal variation of ionization. But in this connection it must be remembered that during the solar eclipse in Canada on August 31, 1932, Dr J. Henderson found that the temporary withdrawal of the sun's ultra-violet light caused a 60 per cent reduction of the ionization in region *E*.

* See *Proc. Phys. Soc.* **42**, 336 (1930).

AN AUTOMATIC RECORDING METHOD FOR WIRELESS INVESTIGATIONS OF THE IONOSPHERE

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ABSTRACT. An apparatus for continuous automatic recording of the equivalent height of wireless echoes returned from the ionosphere is described. The Breit and Tuve method is employed, and both the transmitter and the time base at the receiver are synchronized with the a.c. mains. Some specimen records are reproduced and are used to illustrate the normal diurnal variation of equivalent height. Attention is directed to a common abnormal occurrence of increase of ionization in the lower or *E* region during the hours of darkness, without a corresponding increase in the upper or *F* region. Reasons are given for supposing that this may be due to the ionizing action of storm clouds, as suggested by C. T. R. Wilson.

§ 1. INTRODUCTION

DURING the last few years much information has been obtained about the electrical condition of the ionosphere* by interpreting wireless signals returned from it. The two most useful methods have been the frequency-change method of Appleton and Barnett and the echo method of Breit and Tuve. The results have generally been obtained during rather infrequent "all night runs," in which photographic records of the effective height of the deviating regions were made at intervals of ten or fifteen minutes. The results showed considerable variation from night to night and it is difficult to know what should be taken as typical of the normal night behaviour and what should be taken as abnormal. As is usually the case with geophysical phenomena, it is necessary to observe continuously and daily in order to deduce the normal behaviour. Clearly this is impossible unless an automatic recording apparatus is devised which will work without attention for long periods at a time. A short preliminary account of such an apparatus, for making automatic records of the effective height of the reflecting regions in the ionosphere, has already been given⁽¹⁾. The present paper is a full account of this apparatus together with a discussion of some of the more important results which have so far been obtained from the automatic records†.

* The term "ionosphere" has been suggested to denote the ionized regions of the upper atmosphere.

† While the work now to be described was being carried out some other preliminary reports of similar work have been published^(11, 12, 13).

§ 2. APPARATUS

A method of continuous visual observation, which however did not give automatic records, has already been described by one of the present authors⁽²⁾. The present method is, in principle, the same as the visual method, with some important modifications required to make it yield automatic photographic records.

The Breit and Tuve echo method is employed.

A transmitter is caused to emit short wave-trains of radio-frequency at regular intervals of $\frac{1}{50}$ sec. The duration of the wave-trains, or pulses, is 10^{-4} sec. On arrival at a nearby receiver (about $1\frac{1}{2}$ km. from the transmitter) the wave-trains are amplified and rectified, and the resulting direct current pulses are applied to the vertically deflecting plates of a cathode-ray oscillograph, to the horizontal deflecting plates of which a linear time base, synchronized with the frequency of occurrence of the pulses, is applied. As a result, a stationary pattern of the form shown in figure 1

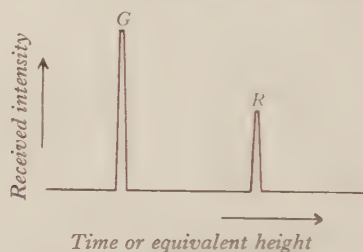


Fig. 1. Appearance of stationary pattern on oscillograph screen.
G, ground wave; R, reflected wave (echo).

appears on the oscillograph screen, owing to persistence of vision. The peak marked G corresponds to the reception of the ground wave, and occurs simultaneously with the emission of the pulse from the transmitter, the time of travel over the distance of $1\frac{1}{2}$ km. between transmitter and receiver being negligible. The peak marked R represents the reception of the signal after it has been returned from the ionized regions of the upper atmosphere, with a time lag of a few milliseconds. From the time lag t the equivalent height h' of the ionized region responsible for the return of the ray may be deduced from the relation $h' = \frac{1}{2}ct$, c being velocity of light in free space. The displacement of the peak R from the peak G is thus proportional to the equivalent height of the deviating region; and if we slowly draw out the pattern in a vertical direction, we shall have immediately a graphical representation of the changes of equivalent height of the deviating region with the time of day.

This is accomplished by forming an image of the pattern on a strip of photographic paper which is moved at about one inch per hour in a vertical direction. The receiver is arranged to saturate for large signals, so that for all intensities of the signal greater than a certain limiting value the tips of the peaks G and R lie on the same horizontal line. An image of this line is formed on a slit covering the photographic film, figure 2.

The chief feature of the present apparatus, as distinct from that previously described, is the method of synchronization of the oscillograph time base with the

transmission of the pulses. For automatic operation, when the apparatus is left to itself for as much as twelve hours at a time, this synchronization must be of a much higher order than that previously achieved by means of a flashing-neon lamp circuit.

Practically perfect synchronization has been attained by using the same a.-c. supply mains to control both the generation of the pulses and the oscillograph time base. The method of control at the transmitter and receiver will now be described.

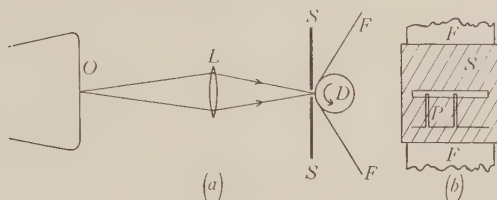


Fig. 2. Optical arrangement for photographic recording. *O*, oscillograph screen; *L*, lens; *SS*, slit; *FF*, bromide paper film, driven past *SS* by the rotating drum *D*; *P*, image of echo pattern formed by *L*.

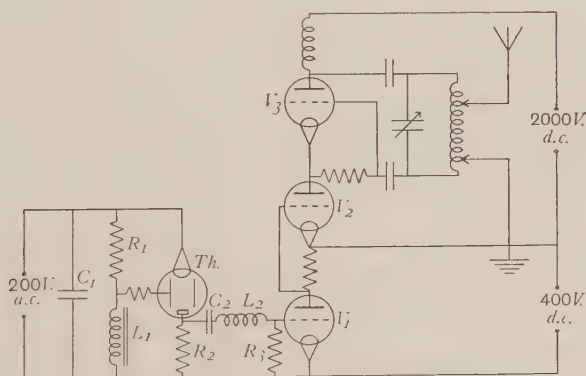


Fig. 3. Circuit of transmitter. C_1 , $0.1 \mu\text{F}$; C_2 , $0.01 \mu\text{F}$; L_2 , 0.16 H ; R_3 , 5000Ω ; *Th*, thyatron; V_1 , pulse-amplifier; V_2 , modulator; V_3 , oscillator.

Transmitter. The transmitter consists of a valve oscillator V_3 , figure 3, arranged in a simple Hartley circuit, controlled by the modulator V_2 in series with it. The grid of V_2 is normally so negative that no anode current can flow in V_2 and V_3 , but at $\frac{1}{60}$ -sec. intervals its potential is made zero for an interval of 10^{-4} sec., thus allowing the transmitter to radiate for this short time.

The required short unidirectional pulses on the grid of V_2 are generated by the thyatron *Th* and its associated circuit. The potentials of the anode and grid of the thyatron are varied at the frequency of the a.c. supply but the potential of the grid is caused to lag about 90° behind that of the anode by means of the inductance L_1 and resistance R_1 , so that the arc strikes when the anode potential is near its maximum value. When striking occurs, almost the whole of this potential is suddenly thrown across R_2 , and excites free oscillations in the circuit $C_2 L_2 R_3$ (completed by *Th* and C_1). The constants are so chosen that the first half oscillation occupies about 10^{-4} sec., and the damping, combined with the excessive negative bias on V_2 , is

sufficient to prevent the further oscillations from having any effect, figure 4. The half oscillation is amplified by V_1 and applied to the grid of V_2 .

The length of the pulses can be altered to suit any purpose by adjustment of the constants C_2, L_2, R_3 whereas the frequency of occurrence is determined by that of the a.c. mains (50 per sec.).

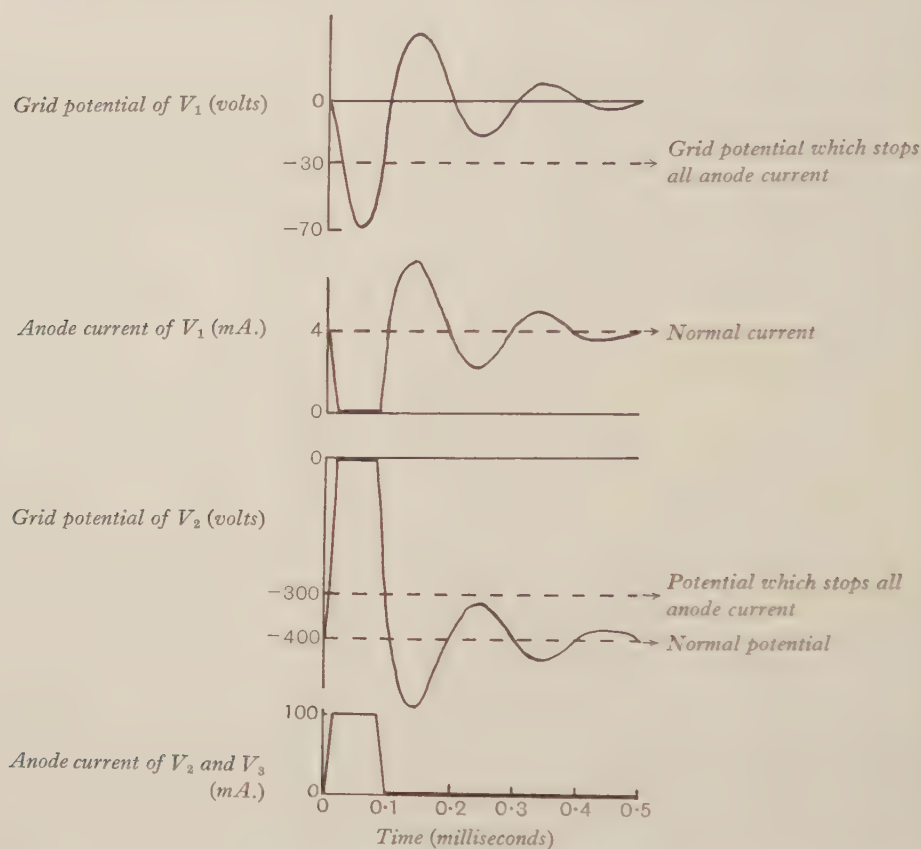


Fig. 4. Production of short pulse from heavily damped oscillation in circuit C_2, L_2, R_3 , figure 3.

All filaments in the transmitter are fed from the a.c. supply and the two h.t. supplies are derived from rectifiers. Also, the grid bias for V_2 is obtained, in effect, from the h.t. for V_1 , so that the whole apparatus runs entirely off the a.c. mains. As it contains no mechanical moving parts it requires no attention whatever, and has often been left running for three or four days continuously. Its starting and stopping may easily be controlled automatically by a time switch.

The transmitting aerial consists of a half-wave horizontal aerial supported at a height of 45 ft., and fed by a single-wire transmission line, adjusted as described by Everitt and Byrne⁽³⁾.

Receiver. The signals are received on a loop aerial, or a horizontal aerial, to which reaction is applied by an entirely independent valve. This considerably increases the

sensitivity without introducing valve "noise." A screen-grid valve in conjunction with a local oscillator is used to convert the signal to a frequency of 110 kc./sec., at which it is amplified by a three-stage resistance-capacity-coupled screen-grid intermediate-frequency amplifier, with band-pass filters for the input and output. The damping and coupling in the filters is arranged to give sensibly uniform amplification for frequencies from 100–120 kc./sec. The output filter feeds a push-pull grid detector stage V_7 and V_8 , figure 5. The advantages of this system are that it responds to very high modulation frequencies, and gives no appreciable amount of intermediate-frequency current in the anode circuit.

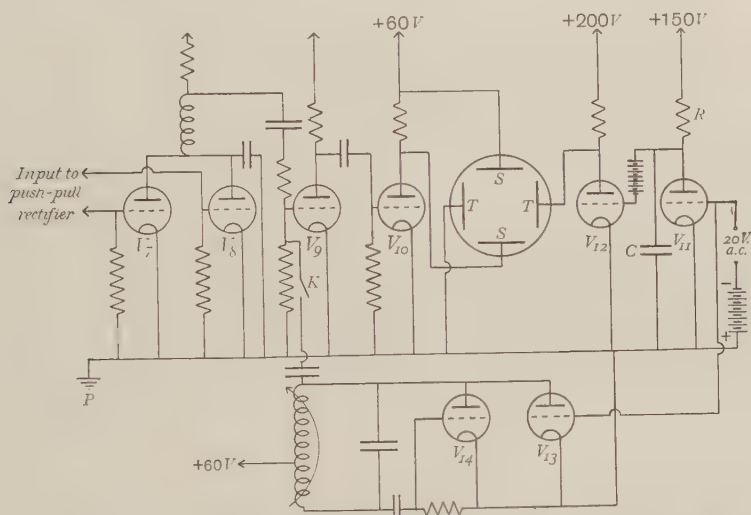


Fig. 5. Circuit diagram of time base and of final stages of receiver. V_7 , V_8 , Push-Pull rectifier; V_9 , V_{10} , L.F. Amplifiers; V_{11} , R , C , Time-base generating circuit; V_{12} , Time-base amplifier; V_{13} , Calibration-oscillation synchronizer; V_{14} , Calibration-oscillation generator.

The rectified output has the form of short unidirectional pulses similar to those used for modulating the transmitter, and after amplification by two resistance coupled low frequency stages V_9 and V_{10} it is applied to the vertically deflecting plates S , S of the oscillograph. The pulses arriving at the grid of V_{10} are negative, so that if the h.-t. supply for this valve is limited to about 60 volts or less a small echo suffices to reduce the anode current to zero and produce the maximum deflection, and any increase of intensity cannot increase the deflection. This is the "saturation device" mentioned above.

Synchronized time base. The synchronized linear time base is provided by a battery of 150 volts charging a $0.1\text{-}\mu\text{F}$ condenser C through a $0.5\text{-}\Omega$ resistance R , figure 5. While it is rising through the first 15 volts the potential across the condenser is practically proportional to the time, and this potential is amplified by V_{12} and applied to the plates T , T of the oscillograph controlling the horizontal deflection. The condenser C is arranged to be rapidly discharged by the valve V_{11} once during each cycle of the a.c. supply, which is the same supply as that controlling the thyatron at the transmitter. This discharge takes place during part of the positive

half-cycle of the a.c. supply to the grid of V_{11} . During the rest of the cycle this grid is so negative that no anode current can flow, negative grid bias being supplied by the battery, so that the rate of charging of C is unaffected by the presence of the valve V_{11} . The time scale can be made as open as desired by suitably altering C and R . When it is made very open the spot traverses the oscillograph screen in a small fraction of a cycle and remains at the end of its trace for the remainder of the cycle, owing to flow of grid current which occurs in V_{11} as soon as the potential across C is equal to the potential of the bias battery; eventually C is again discharged by V_{11} . The time base is calibrated, both for uniformity and for absolute rate of movement, by the low-frequency oscillator V_{11} which has a frequency of 750 per sec., and is synchronized to the 15th harmonic of the a.c. by V_{13} , so that when the switch K is closed a stationary sine wave would be seen on the oscillograph if it were not for the saturating device (i.e. the low anode-potential on V_{10}) which flattens the positive half-cycles. Thus a broken line is photographed on to the film through the slit. These calibration lines are automatically registered on the films for a few seconds at each hour, serving also as timing marks. The period of the broken line is $\frac{1}{750}$ sec., corresponding to an equivalent height of $\frac{1}{2}c \cdot \frac{1}{750}$ or 200 km.

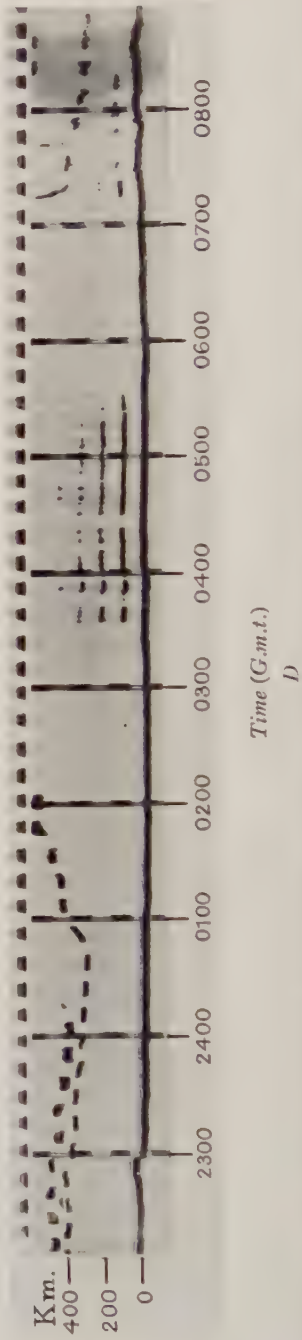
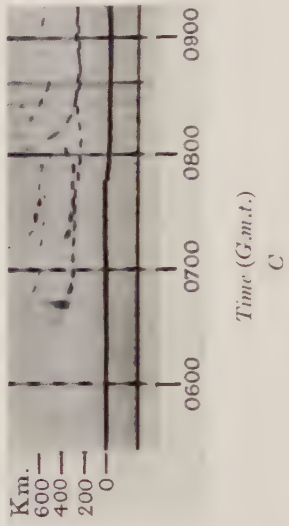
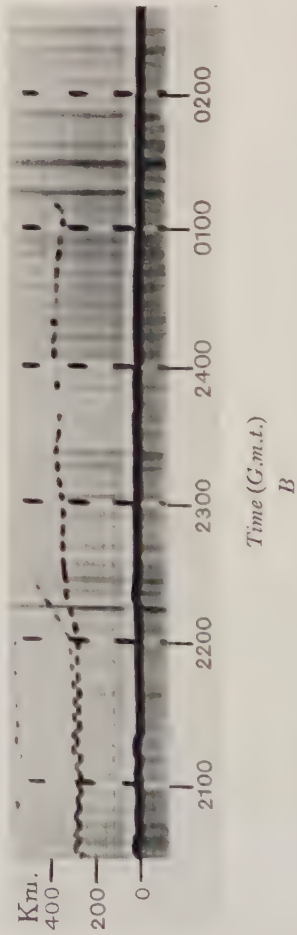
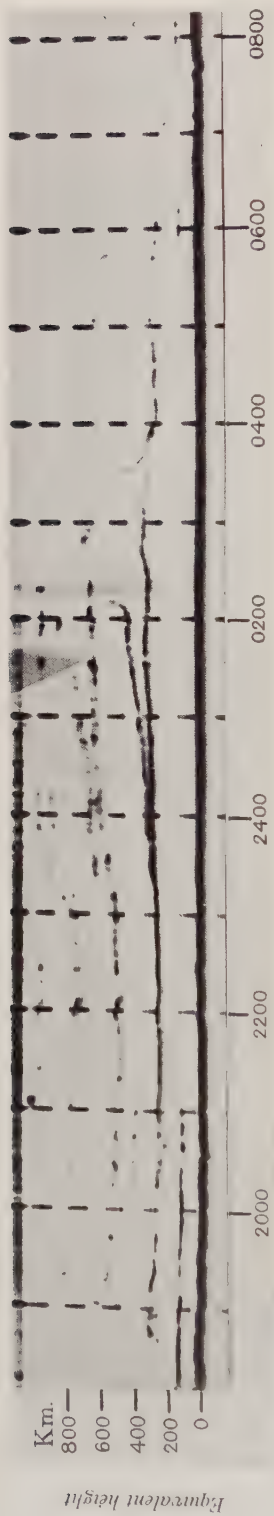
The camera consists simply of a revolving drum round which the film passes immediately behind a slit about 1 mm. wide. An ordinary achromatic camera lens of focal length 5 in. and nominal aperture $f/6$ is used to form an image of the oscillograph pattern on the slit, at a magnification of about unity (see figure 2 above). The film used is 59-mm. paper bromide film. The drum is driven through reduction gears, by a synchronous a.c. motor, to ensure constant speed, so that the film speed is about 1 in./hr.

A Cossor cathode-ray oscillograph with a "photographic-type" screen is employed, and with only 400 volts on the anode it is found to give ample intensity. Under these working conditions the life of the tube, even when run continuously, is found to be very satisfactory.

§3. DISCUSSION OF RECORDS OBTAINED

Normal behaviour. The main object of taking automatic records of the kind described here is to discover what is the normal type of variation of the equivalent height with time of day, and how this variation alters throughout the year. It is also possible that the occurrence of days showing a departure from the normal type may be correlated with the occurrence of other geophysical phenomena, such as magnetic storms. At present the apparatus has not been operating for a sufficient time for yearly variations to be investigated or for extended correlations with other phenomena to be attempted. Sufficient records have been obtained, however, to indicate clearly what is to be considered as normal behaviour, and to point to a very common type of abnormal behaviour.

The plate shows some typical records obtained with the apparatus here described. *A* in the plate is a reproduction of a night-time record obtained on 1st–2nd June 1932 on a wave-length of 100 m. It starts at 1800 G.m.t. and finishes at 0830 G.m.t. The



hours are marked by the vertical calibration lines. These lines also serve as a calibration for effective height. The distance between the ends of two successive dashes in a line corresponds to an effective height of 200 km., as is indicated at the side of the record.

This record gives an example of what we have found to be the normal summer-time behaviour on this wave-length. During the day reflection takes place from the *E* region at an equivalent height of about 120 km. This is seen on the record between 1810 and 2100 hours. Multiple reflections from this region give marks on the record corresponding to equivalent heights which are integral multiples of 120 km. (e.g. between 1850 and 2100 hours). At about 1830 hours reflection from an effective height of about 260 km. becomes apparent (*F* region). An overlap between the *E* and *F* regions usually occurs; if it appears to be absent a stronger transmitter would probably show it up. Finally the *E* reflection vanishes completely and we are left with the *F* reflection alone. The time of this final transfer from *E* to *F* is 2100 hours.

Whenever the *E* region is recorded its height is nearly 125 km.; the extreme limits recorded over two months' observations on this wave-length are 114 and 138 km. This indicates the existence of a sharply defined layer at this height. The effective height does not increase appreciably before the *E* region gives place to the *F* region. On records obtained with this wave-length there is no doubt about the discontinuity of the jump from the *E* to the *F* region. It is always recorded as a discontinuity as shown in *A* in the plate, and has never been recorded as a gradual increase of height. This is important in view of doubts which have recently been cast on the two-layer hypothesis of the ionosphere*.

When reflection from the *F* region is properly established the effective height of the region rises gradually to about 280 km., and then the echo splits into two components having different effective heights, as shown at 2330 hours. The uppermost of these (*F''*) then rises rapidly to about 500 km. and finally disappears at about 0210 hours. In the record of the plate the other component (*F'*) of the split echo remains throughout the night. This occurs only under suitable conditions of wave-length and ionization. A more usual behaviour is for the lower component to rise rapidly to an effective height of about 500 km. and then to disappear an hour or two later than the upper component. This behaviour is shown in record *B* in which *F''* ceases to be reflected at 2220 and *F'* ceases to be reflected at 0110†.

There is next an interval during which no reflection takes place. About 30 minutes before sunrise, reflection of *F'* once more begins, at first at a great effective height (500 km.), but the effective height decreases rapidly. About an hour afterwards *F''* appears and decreases in height to coalesce with *F'*. In *C* in the plate the drop of *F'* is seen at 0645 and that of *F''* at 0715. In this record *F''* appears to rise again and come down finally at 0815. In *A* in the plate the drop of *F''* occurs at

* It is, however, noticeable that, with records obtained recently on a wave-length of 175 m., there are distinct signs of a gradual change in effective height from the *F* region to the *E* region near sunrise.

† In *B*, *C* and *D* in the plate the dotted nature of the records is due to the fact that a special polarized receiver was used, which was switched so as to receive only *F'* or *F''*. The dashed line at the top of the record is made by an indicating device. An account of this receiver will be published elsewhere; for the purposes of the present paper the broken lines should be imagined to be completed.

0330 hours. An hour or two after sunrise, which here occurred at 0350, the effective height of the F region has dropped to about 260 km., and the E region at 125 km. begins to appear, and remains throughout the day.

The most interesting feature of this normal daily behaviour is the splitting of the echo from the F region. This behaviour is similar to that previously described by Appleton and Builder⁽⁴⁾ and ascribed by them to the effect of the steady magnetic field of the earth. It is supposed that, during the night, the electron-density in the F region decreases until it is no longer sufficient to reduce the refractive index to zero for one of the elliptically polarized waves, which may be propagated, according to the magneto-ionic theory, so that this wave will cease to be reflected (disappearance of F''). There will still be enough electrons, however, to reflect the other elliptically polarized wave F' , and the recombination has to proceed for about another hour before this in turn ceases to be reflected.

It may be shown that the refractive indices μ_0 and μ_e become zero for the two waves for the following values of critical electron density N .

$$\mu_0 = 0 \text{ when } N = \frac{3}{2} \frac{mp^2}{4\pi e^2} = N_0$$

$$\mu_e = 0 \text{ when } N = \frac{3}{2} \frac{mp^2}{4\pi e^2} \left\{ 1 - \frac{p_1}{p} \right\} = N_e.$$

N_0, μ_e, N_e Here μ_0 and N_0 refer to the ordinary wave, and μ_e and N_e to the extraordinary wave,

m, e are mass and charge of electron,

p = the angular frequency of the wave, and

$p_1 = He/mc$ where H is the earth's field in gauss.

p
 p_1, H
 t The time interval t , between the vanishing of F'' and F' therefore gives us the time which must elapse for the maximum electron-density in the ionosphere to change from N_0 to N_e . If this decrease is due to the recombination of positive and negative ions with a recombination coefficient α it may be shown that

$$\frac{1}{N_e} - \frac{1}{N_0} = \alpha t.$$

Thus from the observed time t taken from the records it is possible to calculate α . The results so far obtained show that the time interval t is somewhat variable. The variability may be due to the fact that the maximum of ionization does not occur at the same height from night to night, so that the pressure and hence the recombination coefficient is different on different nights, or it may be due to the fact that, even during the night, some hitherto unknown cause of ionization is present*. The order of magnitude of this variation is from 20 min. to 3 hr. and the mean time is about 1 hr. for a wave-length of 100 m. This gives $\alpha = 1.5 \times 10^{-9}$. In view of the uncertainty of our knowledge of the atmospheric pressure at these heights and of the magnitude of the recombination coefficient at low pressures, a comparison of this value with the theoretically predicted value would not seem to serve any useful purpose.

* It may be that effects of this kind are correlated with the abnormal behaviour described on the next page.

However, this value agrees fairly well with that of 9×10^{-9} found by Eckersley by quite a different method*.

An interesting feature of the splitting phenomenon is the very great effective height which the two echoes attain before they ultimately vanish. In interpreting this phenomenon it must be remembered that the effective height depends on the group velocity of the waves. In the presence of a steady magnetic field the group velocity is related in a complicated way to the phase velocity, but it can be shown that even in the general case the group velocity is approximately equal to the reciprocal of the phase velocity. At the highest point of the trajectory, where the ionization-density reaches the critical value, the phase velocity is very great and the group velocity is therefore very small. The total time of travel of the group is therefore determined, to a large extent, by the geometrical distance over which the electron density is nearly critical. When the wave is just on the point of penetrating the *F* region entirely, reflection takes from a point where *N* is a maximum and the ionization-gradient is gradual. A considerable length of the geometrical path is then traversed with the ionization-density nearly critical and with the group velocity abnormally small, so that the total time is considerably lengthened. When there are more than enough electrons to reflect the wave, reflection takes place from a point where the gradient of ionization is sharper and only a short length of the geometrical path lies in the region of critical ionic density, and hence of small group velocity, so that the group retardation effect is less noticeable. There must, of course, be an increase of geometrical path in going from the place where the ionization is not critical to the place where it is critical; but this increase is probably much smaller than the apparent increase due to the low group velocity. The present experimental method does not enable this increase in geometrical height to be determined.

Abnormal behaviour. It has been pointed out above that on most records for these wave-lengths the *E*-region echo disappears and gives way to the *F*-region echo round about sunset, and appears again round about sunrise. This we call the "normal" behaviour. On several occasions, however, it has been observed that after the *E* echo has disappeared in the usual way near sunset, it has reappeared for a few hours round about midnight†. It has then disappeared again and reappeared at sunrise in the normal way. In the typical cases there is always an interval between the disappearance of the normal *E* echo at sunset and the appearance of the abnormal echo. It appears that this must imply an increase in the *E*-region ionization during the midnight hours. This cannot be due to any ionizing influence which is propagated rectilinearly from the sun, for sunset at a height of 100 km. occurs 40 min. after ground sunset. Now from the correlation between sunrise and the incidence of the normal *E* echo in the early morning, it is fairly certain that the normal *E*-region ionization is produced by something which is propagated rectilinearly from the sun,

* Although, for comparison with Eckersley's value, we have here calculated the recombination coefficient α , it is by no means certain that the electrons are lost by recombination; they may be rendered ineffective by attachment to neutral molecules.

† Similar observations have been recorded by Appleton and Naismith⁽⁶⁾, Schafer and Goodall⁽⁷⁾, and Ranzi⁽⁸⁾.

either ultra-violet light or uncharged corpuscles. There thus appears to be reason for supposing that the abnormal *E* ionization and the normal *E* ionization are due to different causes. This supposition receives support in another way from our records. The incidence of the normal *E* region at sunrise is always closely correlated with an increase of the ionization of the *F* region. This is shown, for example, in the record *A* in the plate in which the appearance of the *E* region is preceded by a joining together of the two components of the split *F* echo and by a marked decrease in the effective height of the *F* echo as a whole. It looks as though the ionization of both the *F* region and the normal *E* region is increased simultaneously by an ionizing agent propagated rectilinearly from the sun. It is quite different, however, with the abnormal *E* region, as it appears from some of the records that the increase of ionization which is responsible for the appearance of this region is not accompanied by a corresponding increase of ionization in the *F* region. Thus in *D* in the plate reflection of the *F* echoes ceased at 2415 and 0205 and began again at 0715 and 0745 in the ordinary way. It seems fairly certain that between 0205 and 0715 there were not enough electrons in the *F* region to produce reflections. It is, however, found that between 0330 and 0530 reflection took place from the *E* region with such intensity that four reflections were recorded, and this without any reappearance of the *F* region echo. The *E* region had therefore been ionized to a density sufficient to give an echo, without any apparent increase in the *F*-region ionization. There seem to be two possible explanations of this occurrence. Either the abnormal *E* region is ionized by some influence which comes from outside the atmosphere and is capable of passing through the *F* region without ionizing it, or the ionization is produced by some source situated below the *E* region, so that its influence will be felt by the *E* region more strongly than by the *F* region. If the first of these alternatives is supposed correct it must further be supposed that the ionizing agent is not propagated rectilinearly from the sun. A stream of charged particles coming from the sun and deviated by the earth's magnetic field so as to reach the dark side of the earth is a possible source of ionization of this kind. Such a stream would produce the most intense ionization near the end of its range, and so might ionize the *E* region without ionizing the *F* region. It has been supposed that the ionization due to such a stream is responsible for magnetic storms, and so it seems reasonable to look for a correlation between magnetic storms and the occurrence of the midnight *E* region. Over the limited period during which the apparatus has been running it has not been possible to find any such correlation.

When we discussed with Prof. C. T. R. Wilson the second possibility, a terrestrial source of ionization, he pointed out to us that such a source was probably available in thunder clouds and shower clouds. In a discussion at the Physical Society⁽⁵⁾ on ionization in the atmosphere he stated that "if there were no already existing conducting layer there is little doubt that a thunder-cloud would itself cause ionization in the upper atmosphere." He discussed two possible mechanisms by which the thunder cloud or shower cloud might produce ionization in the ionosphere. (1) The electric field due to the electric moment of the cloud may itself produce ionization at heights above 80 km. where the pressure is sufficiently low; and (2) by

its accelerating action on charged particles the electric field of a thunder cloud may produce very penetrating corpuscular radiation, which in turn may produce ionization in the ionosphere. It appears that clouds of this type may provide just such a terrestrial source of ionization as our experiments seem to indicate. In looking for a correlation between the occurrence of the abnormal *E* region and thunder clouds or shower clouds it is important to remember the following points stressed by Prof. Wilson. "The vertical electric force at a point (at a height of 60 km.) may not be due solely to a thunder cloud vertically below it; all thunder clouds within a considerable area will contribute to it." "It is possible that the electrical field of a large rain cloud, which would not be regarded as a thunder cloud, may be strong enough to cause discharge in the atmosphere above it."

The suggestion that storm clouds may be responsible for ionizing the lower region during the hours of darkness receives further support from Ranzi's observation⁽⁹⁾ that the appearance of the "abnormal *E* region" is correlated with the occurrence of meteorological low pressure conditions. Our results entirely confirm his observations and seem to show an even closer correlation with thunderstorms and heavy rainfall. The observed fact that the associated storm centres may be some distance removed from the place of observation, suggests the possibility that the extra ionization produced by the storm may spread by diffusion in the *E* region.

It is of special interest to notice that the echo from the abnormal *E* region in the night is much less absorbed than that from the normal *E* region in the daytime, as is evidenced by the large number of multiply reflected echoes which are received at night with sufficient intensity to be recorded. In spite of this, however, both types of reflection occur at approximately the same height. The explanation of these facts may be along two different lines; either the difference in absorption is due mainly to a change in the temperature of the ionosphere between day and night (the absorption depending on the collisional frequency and hence on the temperature), or the temperature remains roughly constant and the ionization-distribution changes, so that there is less ionization in the night-time in an absorbing region situated beneath the deviating region. The latter type of explanation would agree with a suggestion previously made, on quite different evidence, by Appleton and Ratcliffe⁽¹⁰⁾.

§4. ACKNOWLEDGMENTS

We wish to express our thanks to Prof. C. T. R. Wilson for the many helpful discussions we have had with him, and for the characteristic readiness with which he has always put his deep knowledge of thunderstorms at our disposal. We also wish to thank Mr L. B. Turner for providing us with an ideal situation for the erection of our transmitter in the Cambridge University Engineering Laboratory.

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DISCUSSION

Mr R. NAISMITH. The rather sudden appearance of triple echoes from the *E* region, shown as *D* in the plate, suggests a very rapid increase in ionization. I should like to ask the authors whether they consider that the electric field due to the electric moment of a cloud and its accelerating action on charged particles could, under comparatively normal conditions (i.e. with no well-marked storm centre within a considerable area of the observing site) in these latitudes at mid-winter produce such a sudden increase in the ionization of *E* region. It is perhaps unfortunate that the record chosen to illustrate the appearance of abnormal *E* echoes was taken on December 14-15, 1932. On that particular occasion the largest magnetic disturbance of recent months took place, which strongly suggests that the abnormal ionization in *E* on this occasion was due to increased magnetic activity. It will be appreciated that the time of sunset (quoted at the foot of page 407) at a height of 100 km. will vary with the declination of the sun.

Mr F. E. LUTKIN. Mr Naismith has expressed doubts whether the record of abnormal night ionization in *E* layer, shown in record *D* for December 14, 1932, should be associated in midwinter with thunderstorm activity, and I must confess to surprise at finding a similar abnormality in *E* during the extensive anticyclonic conditions of January 25. Ranzi states that the majority of his cases of midnight *E* are associated with a depression at or to the north of his Italian station, and the authors express agreement with his findings. May I, therefore, suggest a mechanism which will hold good for Ranzi's general condition and also for the winter cases quoted above. A depression to the north of Italy is associated with a meteorological front, a region of sharp temperature-contrast, where warm moist air is being forced up over cold air extending out over the Mediterranean. Because the area is over

sea, associated electrical phenomena will probably occur chiefly during the night hours. Similar low-pressure areas to the north or north-west of the British station will have similar fronts extending over the near Atlantic. Although no marked depression lay over the north of England on either of the dates referred to above, extensive frontal systems existed in the north Atlantic, and since the frequency of thunderstorms over sea areas varies very little with season there would be equal probability of finding night *E* from this cause all the year round, and doubts in connexion with the winter season disappear. I therefore suggest the possibility that midnight *E* indicates an electrically active front over the sea within some 1000 km. of the recording station. Whether the longitudinal distance of the front from the station bears any relationship to the hour at which *E* is recorded might be worthy of investigation.

Dr M. TAYLOR. The very elegant recording method described in the paper is likely to prove extremely valuable in wireless investigations of the ionosphere. On the theoretical side I think it would be very helpful if the authors would indicate the principles on which their theoretical discussion of group velocity is based. Prof. Appleton has discussed the meaning of group velocity in an ionized medium without the complications of two refractive indices introduced by an external magnetic field, and we are accustomed to the independence of the ordinary and extraordinary waves in a doubly refracting crystal in which the two refractive indices are constant. But in a medium of varying ionization under the action of an external magnetic field the refractive indices change continuously, and the ordinary and extraordinary waves, when they reach a place of different ionization, will each split up and produce an ordinary and extraordinary wave with refractive indices corresponding to the new ionization-density. Thus the ordinary and extraordinary waves cannot, without further investigation, be regarded as pursuing independent paths, as they may in a doubly refracting crystal, and this resplitting complicates the meaning of group velocity in the ionosphere. Hitherto I have looked upon the experimental results regarding split echoes of long delay, which the authors describe, as giving empirical evidence as to the nature of group velocity, which awaited theoretical explanation, rather than as experimental confirmations of properties of the medium predicted by theory. It would be very interesting if the authors could tell us shortly how to show that "the group velocity is approximately equal to the reciprocal of the phase velocity."

Dr D. OWEN. I think the Society is to be congratulated on the succession of interesting papers on the ionosphere recently presented by several workers in this field. The progress made in the technique of measurement is indeed remarkable, and has been duly followed by a stream of very interesting information.

As to the effect of thunderstorms on the ionization of the *E* layer, is it considered possible that there is an actual transference of ions, from a layer 10 km. from the earth to the *E* layer at, say, 100 km.? If so a considerable time-lag would seem to be required, and may perhaps have been observed.

AUTHORS' reply. It does not seem to us that the sudden appearance of an echo from the *E* region necessarily implies a *rapid* increase of ionization, as suggested by Mr Naismith; the appearance of the multiple echoes simultaneously probably indicates merely an absence of absorption, as is mentioned in our last paragraph. The time given for sunset at 100 km. is only meant to be a rough value. We are aware of the effect of the sun's declination, and are obtaining some interesting results in this connexion by recording the time of commencement of the *F* echo throughout the year.

With reference to the remarks of several speakers as to the possible mechanisms of ionization of the abnormal *E* region, it will be clear from the paper that at present we are only suggesting possibilities, and a complete discussion must be postponed until further data are available. If, however, the phenomenon is to be ascribed to occurrences in the lower atmosphere, it seems reasonable to suppose that rain-cloud action of the type suggested by C. T. R. Wilson is the chief mechanism, and that any state of affairs which leads to the production of this type of cloud may be correlated with the phenomenon. In particular, if meteorological fronts prove to be correlated with the effects we have described, it must be in virtue of the fact that the appropriate type of cloud occurs in the frontal region. We are well aware of the importance of such fronts, and are engaged in exploring their connexion with our phenomenon; Ranzi himself* drew attention to their effect on wireless signals.

We are aware that the record of December 14-15 coincided with an intense magnetic disturbance as well as with an extremely deep meteorological depression, and so far as this record alone is concerned either of our explanations may be the correct one.

Dr Taylor's remarks raise two points in which we have been specially interested. The first is that a single pulse sent into the ionosphere is often returned as two discrete pulses and not as a single broadened pulse such as might be expected from the "splitting and resplitting" argument which she outlines. Although we consider that there are theoretical reasons for expecting this behaviour when the properties of the medium change slowly with distance, it is sufficient to take it, from the point of view of this paper, as an experimental fact as she suggests. It then seems necessary to suppose that the original group is propagated, at each point in the medium, as two independent groups. This brings us to the second point: "What is the group velocity in a magneto-ionic medium with constant ionic density?" In a uniform medium of this kind the ordinary conception of group velocity holds, and the group refractive index μ' is given in terms of the phase refractive index μ by the usual expression

$$\begin{aligned}\mu' &= \mu + p \frac{d\mu}{dp} \\ &= \frac{1}{\mu} \left\{ \mu^2 + \frac{1}{2} p \frac{d}{dp} (\mu^2) \right\},\end{aligned}$$

where p is the angular frequency of the wave. In the absence of the magnetic field

* *Nuovo Cimento*, **8**, 98 (1931).

the factor in the bracket is unity and $\mu' = \mu^{-1}$. In the presence of the magnetic field the magnitude of the factor can be estimated under any given conditions by sketching the curve of μ^2 against N for the frequencies p , $p + \Delta p$ and $p - \Delta p$ and performing the differentiation numerically. We have developed simple rules for sketching the curves of μ^2 against N , and using these we find, for the wavelengths here considered, that the factor is of the order unity except near $\mu = 0$, and then it becomes large. Near $\mu = 0$, however, the factor μ^{-1} itself becomes considerably larger and dominates the expression, so that, to a first approximation, we may take μ' equal to μ^{-1} for all values of μ ; and in particular μ' is very large when μ is small, which is the deduction used in the paper.

In reply to Dr Owen: we have not yet sufficient data to decide which of the mechanisms suggested by Prof. C. T. R. Wilson is most important in the ionization of the ionosphere by storm clouds. It seems fairly certain that an actual transference of ions from the cloud to the E region is not so important as the production of ionization in, or near, the E region by the field of the cloud or by penetrating radiation produced by the cloud.

THE TRANSMISSION OF HEAT THROUGH FABRICS*

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ABSTRACT. The results obtained in earlier papers^(1,2) have been analyzed to determine the effects of air permeability and of perforations on the thermal insulating properties of fabrics. It is subsequently shown that there is a heat-interchange between the convection currents and the fabric which is important in considering the flow of heat through such insulators.

§ 1. INTRODUCTION

A LARGE number of thermal insulators, such as textile fabrics, are permeable to air and allow the transmission of radiant heat through their interstices. Such insulators generally have not a smooth surface, due to the interlacing of the constituent yarn, and thus the measurement of surface temperatures as ordinarily understood is not possible. Because of their compressible nature, their power of thermal transmission is changed by such compression as would occur during testing in the usual type of thermal-conductivity apparatus.

The thermal insulating properties of fabrics have been studied by the author^(1,2) by means of an apparatus involving two horizontal concentric cylinders maintained at different constant temperatures, in which the sample of fabric under test was supported as freely as possible in the resulting annular space in the form of an intermediate concentric cylinder. Guard rings eliminated end effects, and due experimental precautions were taken.

The following were the dimensions of the apparatus. In every case a length of 25.0 cm. is used. The diameter of the hot inner cylinder is 5.50 cm. giving an area of 431 cm². The outer boundary is 12.70 cm. in diameter internally and, unless otherwise stated, is considered as blackened.

The results obtained for a very wide range of fabrics were expressed in terms of *thermal insulating value* (T.I.V.), i.e. the percentage reduction of heat flow caused by the fabric under the specified test conditions. The thickness of each fabric under a definite load per square centimetre was measured by means of an instrument previously described⁽³⁾. The air permeability and percentage hole-area (light transmission) were also determined. The thermal insulating values when plotted against thickness presented a series of points which were found to lie on a band on either side of a straight line which did not pass through the origin. This diagram was taken as a basis of discussion and it was suggested that the deviation of the points from the mean line is due to the differences in air permeability and light-

* This paper forms part of a thesis approved for the degree of Doctor of Philosophy in the University of London.

transmission, which allow different amounts of air and of radiant heat to pass through the fabrics. In the previous paper on thermal transmission of fabrics this suggestion was not applied to the results obtained except in a general way. The object of the present paper is to test the validity of this suggestion in the light of the results obtained and to discuss the theories which may be involved.

It should be noted that, throughout, hygroscopic equilibrium is assumed, and thus no heat is transferred by evaporation at the warmer, and condensation at the cooler, places.

§ 2. EXPERIMENTAL RESULTS

The results recorded for various fabrics in the previous papers have been examined to find the effects of air permeability and percentage hole-area (as measured by light-transmission) on the total heat-flow. As the fabrics were different in structure, material, emissivity and surface roughness (which is thought to cause increased convection from the surface), it is not to be expected that the thickness, hole-area and air permeability cited above will explain the whole of the phenomena of heat-transmission. This conclusion was confirmed for single results or groups of results, and no definite conclusions could be reached in this way.

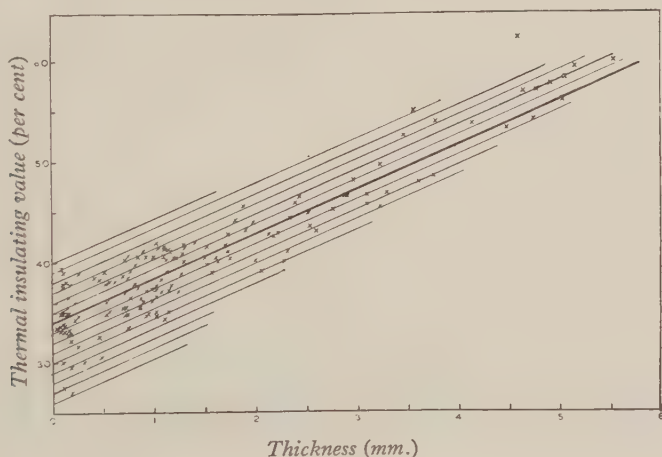


Figure 1.

As a fairly large number of results were available (about 130) it was decided to use a statistical method for their treatment. It was very necessary, however, to exercise caution in avoiding the employment of groups containing very small numbers of results. The thermal insulating values were plotted against the thickness, and this diagram, figure 1, formed the basis of the analysis. The best straight line was drawn through the points by the simple method due to Awbery⁽⁴⁾. This method consists essentially in finding the "centre of mass" of all the points and then determining what might be called the "mean gradient"* of the line joining

* For the actual process used in finding the mean gradient, reference should be made to Awbery's paper.

each point to the centre of mass. The "best" line has a gradient equal to the value so obtained, and passes through the centre of mass.

Such a line was drawn on the diagram and a series of other lines parallel to it were added at regular intervals, dividing the points up into groups according to their distance from the best line. If the theory suggested above for heat flow and its dependence on air permeability and hole-area is correct, the fabrics corresponding to the points lying in the lowest groups should have the highest permeability and percentage hole-area, and *vice versa*. Each point was therefore identified to its corresponding fabric and recorded in its appropriate group together with the full particulars of the fabric represented by it.

On taking the mean permeability and light-transmission for each group, the general trend of the results could be seen. It was found that intervals of 1 per cent in thermal insulating value gave erratic means and were therefore too small, but intervals of 2 per cent were satisfactory. Larger intervals would have given too few groups.

The best line may be considered as the relation between thermal insulating value and thickness for average fabrics. The other lines parallel to it relate to fabrics which, for a given thickness, differ in thermal insulating value from an average fabric by a certain number of units; i.e. they are fabrics in which there is a certain difference in heat-flow. It is, therefore, possible to assign to each group a mean excess heat-flow above the average which will henceforth be referred to as *excess heat flow*. It is a convenient quantity to consider in interpreting the physical meaning of the results obtained.

Table 1 shows the results of this analysis. The figures shown in brackets are the mean values given by such small groups that no significance beyond general trend can be attributed to them.

Table 1

Interval (in thermal-insulating-value units)	Excess of heat-flow above mean (W.)	Mean air permeability of all points within the interval (cm^2 per cm^2 per sec. per cm. head of water)	Mean percentage light-transmission of all points within the interval
- 8 to - 6	.538	(.915)	(36.2)
- 6 to - 4	.385	680	17.9
- 4 to - 2	.231	230	8.4
- 2 to 0	.077	104	4.2
0 to 2	-.077	99	4.2
2 to 4	-.231	79	4.2
4 to 6	-.385	(28)	(4.6)

The full-line curves in figures 2 and 3 show the relation between excess heat-flow and air permeability and percentage hole-area respectively. They show that there is a definite increase in heat-flow with increase in these quantities. Thus the early suggestions as to the curves of the variations of the points from the mean line on the (thickness, thermal insulating value) curve were feasible.

An attempt to analyse the result in the reverse way, i.e. by taking groups of values of the air permeability and percentage hole-area and finding the mean error from the best line for each group, was not so useful as it gave one or two very large groups and several very small ones. This method therefore is not considered here.

The most distinctive feature of the curves is that each is made up of two parts of very different gradients. This is especially marked in figure 3, which shows that for several groups the average percentage hole-area is practically constant. The increase in heat-flow in this part must therefore be due wholly to the air currents through the fabrics. It will be seen from figure 2 that in this region there is approximately a linear relation between the excess heat-flow and air permeability.

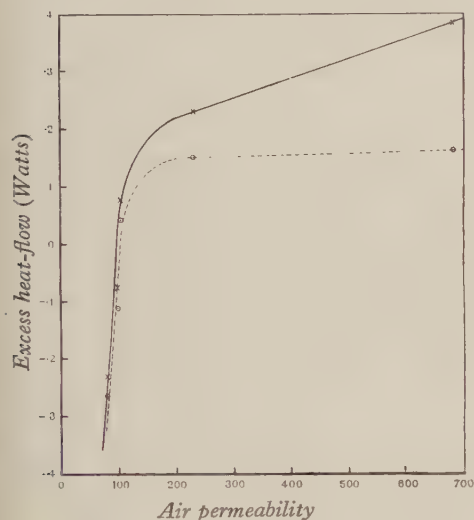


Figure 2.

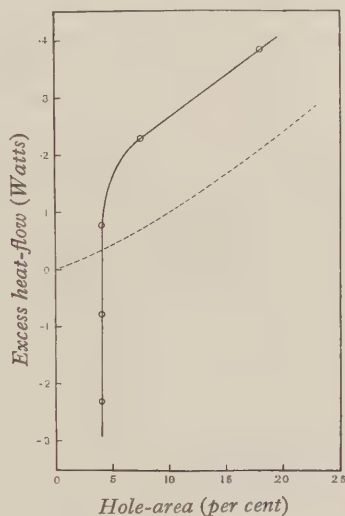


Figure 3.

It is necessary to explain the significance of the two parts of the curves and the very pronounced change in gradient, which is too great to be due to experimental errors. It is possible to suggest many mechanisms, but the one given below is the only one by which the author has been able to obtain any explanation of the bends in the curve.

§ 3. DISCUSSION OF RESULTS

In the curves for excess heat-flow against air permeability and hole-area respectively, shown in figures 2 and 3, the effects of air-flow and radiation are combined. It is necessary to separate these two effects. The radiation effect will be treated first, as its effects are more simply calculated and hence the air permeability effect may be deduced.

Suppose a hot body is giving off R units of heat per second towards a cold boundary in front of which is a screen pierced with holes amounting to n per cent of its area, and which has a temperature intermediate between the two. It is then obvious that $nR/100$ units of heat per second are transmitted directly through the

R
 n

holes in the screen to the cold outer boundary. Gregory⁽⁵⁾ has shown that as regards radiation a fabric behaves very nearly like a black body, so that for insulators such as these the small amount of radiant heat which may pass through the fabric, suffering one or two reflections in the process, is not significant. Attention may therefore be confined to direct transmission through the holes. The heat transmitted is proportional to the percentage hole area n , and R will subsequently be shown to be also a function of n . As far as could be ascertained the literature contains no records of work, either theoretical or practical, on radiation from a body through a perforated screen to a cold boundary. The following method of consideration has therefore been adopted. If a hot body is radiating heat in an enclosed space, the distance of the colder boundary walls of such enclosure from the radiating body has no effect on the quantity of heat radiated. The outer boundary can therefore be moved for purposes of calculation to any convenient position. In the case of a perforated screen and the outer boundary considered here, it is possible to bring them into coincidence and so produce a composite boundary with patches at screen and boundary temperatures. If the absolute temperature of the hot body be T_0 , that of the inner surface of the screen T_1 , and that of the cold outer boundary T_2 , the conditions obtaining will be that $(100 - n)$ per cent of the surface is at T_1 and n per cent at T_2 .

Two methods may be considered for calculating the radiation from this surface. The radiation might be calculated from the fourth power of the mean temperature. This is incorrect since each element of area radiates according to its own absolute temperature and not in proportion to the mean temperature of the total area. It is therefore necessary to consider each infinitesimal area separately, and the individual radiations should then be summed and meaned. In the present case this process is simple and gives

$$R = SE\sigma \left\{ T_0^4 - \left(\frac{100 - n}{100} \right) T_1^4 - \left(\frac{n}{100} \right) T_2^4 \right\} \quad \text{.....(1)}$$

E, σ, S

where E is the emissivity, σ the Stefan-Boltzmann constant and S the area of the radiating surface. This expression (1) assumes that the insulator and the outer boundary are both equivalent to black bodies, which assumption is true in this case. Thus the heat transmitted directly by radiation through the holes is

$$\frac{nR}{100}, \text{ which } = \frac{nSE}{100} \sigma \left\{ T_0^4 - \left(\frac{100 - n}{100} \right) T_1^4 - \left(\frac{n}{100} \right) T_2^4 \right\}.$$

The two unknowns in this equation are E and T_1 . It is possible to obtain approximate values of the emissivity by a consideration of the heat-loss by radiation from the bare cylinder, which can be calculated from Stefan's law. Taking σ as $1.36 \cdot 10^{-12}$ cal./cm²-sec. and the cold boundary at 12° C., i.e. according to the conditions of experiment, table 2 has been calculated for a black inner cylinder.

Table 2

Temperature-difference (° C.)	0°	5°	10°	15°	20	25°	30°
Temperature of inner cylinder (° C.)	12°	17°	22°	27°	32°	37°	42°
Heat-loss, calculated (W.)	0	1.17	2.39	3.68	5.05	6.48	7.96

Thus for a temperature-difference of 25°C ., the heat lost by radiation if the cylinder was black was 6.48 W. Curves were given in the author's previous papers for total heat-loss from the cylinder when the cold receiving surface was bright, and alternatively when it was black. At 25°C . the curve shows that the difference between the total heat-flows in the two cases amounts to about 2.7 W., and this must all be due to radiation; and since only radiation is affected by surface conditions this factor accounts for at least 2.7 W. It is not likely to account for much more, so that the emissivity may be taken as $2.7/6.5 = 0.42$. This is of the same order as that obtained for paints with metallic bases⁽⁶⁾.

The only unknown quantity left from the equation above is T_1 , and since it is unlikely to be below 25°C . or above 35°C ., a mean value of 30°C . or 303°K . has been taken for purposes of calculation. The difference from such a value is relatively small. The calculated values are shown by the dotted curve in figure 3, which indicates nothing which would account for the bend in the excess-heat-flow curves.

Attention must therefore be directed to the (excess heat flow, air permeability) curve in figure 2. Since the relationship between the radiation effect and percentage hole-area is known, the radiation through the holes for each group can be calculated from the mean percentage hole-area. By subtraction of this value from the excess heat-flow, the effect due to the air permeability can be found. The result is shown on the dotted curve in figure 2. The distinct bend of the curve still remains, and it is therefore in this part of the excess heat-flow that the explanation of this bend is to be sought.

In considering the flow of heat by convection through the fabric, it is first necessary to know the general form of the air current through the fabric. A model was therefore made consisting of two concentric tubes of the same diameter as the original apparatus, but only 8 cm. long. One end of the pair of tubes was closed by a metal plate and the other by a glass window, all joints being made airtight. The fabric was supported on a cylindrical metal framework which was mounted concentrically with the other cylinders and made air-tight contact at its ends with the metal plate and window respectively. Thus there was no passage for the air between the inner and outer annular spaces except through the fabric. After the inner cylinder had been heated from the inside for some time, puffs of smoke were admitted at various places near the inner cylinder, and the paths of the smoke were carefully watched. The general impression of the observations is shown diagrammatically in figure 4. The continuation of the smoke streams in the space between the inner cylinder and the fabric could not be very well observed, owing to the fact that there is another convection system of a similar nature in this space, and it is impossible to separate the two.

It has been suggested that a convection system of the type illustrated could not be obtained if the inner and outer cylinders had uniform temperatures throughout their circumferences. While this may be strictly true, the differences in temperature between the various parts of the cylinders must be very small owing to the thickness of the walls and the methods of heating and cooling.

As the air percolates through the fabric, there is a transfer of heat between the air-stream and the solid insulator, and it will now be shown that the bend in the curve is due to this interchange of heat. In the attempt to do this mathematically two difficulties are at once encountered. One is that the exact law of heat-transfer from the air to the solid is not known. The other is that, if a simple law be assumed, the number of variables depending on each other renders the integration difficult. The present discussion is therefore not quantitative. It has been shown that the convection results in an outward flow of air through the upper part of the fabric, which is in the form of a horizontal cylinder in the experiments, and an inward flow through the lower part. It may also be safely assumed that the temperature of the fabric is between that of the cool air coming to the hot cylinder from below and that of the heated air leaving the cylinder at the top. Considering first the upper half, we note that the effect of the heat-transfer here is to cool the air and thus to diminish the total heat flowing out from the hot cylinder. In the lower half,

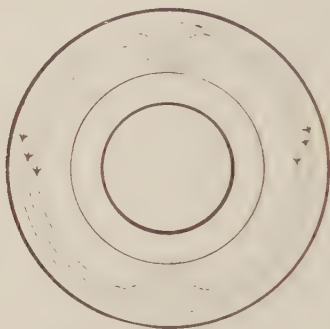


Figure 4.

the effect of the heat-transfer is to heat the air stream. This means that there is an increase in heat flowing *towards* the hot cylinder, which again is equivalent to a diminution of heat flowing away from the cylinder. It is clear, therefore, that in both halves of the fabric, as the transfer of heat between the air stream and fabric becomes larger, the total heat-flow will become smaller.

The effect of the heat-transfer on the fabric must also be considered, since the total heat-flow depends to some extent on the temperature of the fabric. The consideration given above shows that the upper half of the fabric will be heated and the lower half cooled as a result of the heat-transfer. The two effects will thus neutralize each other to a large extent.

In the case of textile fabrics, those of open texture (i.e. having a large number of holes and therefore a high value for light transmission) possess high permeability for air. The stream of hot air can thus pass directly through such holes without impedence, and hence the transfer of heat from the hot air to the insulator will be small, since the facilities for heat-transfer will be small. As the degree of such perforation decreases, as indicated by decreasing values of percentage hole-area, there comes a critical point when direct transmission of air is no longer possible,

so that in percolating through the material the air will follow a more or less devious path through the interstices. In this way the heat-transfer from moving air stream to fabric is greatly increased. If the texture is made still closer and therefore the hole-area still further decreased below this critical value, then obviously the heat-flow through the fabric should be less than that indicated by extrapolation from the part of the curve showing higher permeability values. It would therefore be expected that these two sets of conditions, more or less gradating into each other, which are obtained on either side of the critical value, will be indicated in the curve for air permeability and excess heat-flow by a distinct bend. In figure 2 such a bend actually does occur and is seen in the neighbourhood of 150 cm^3 per sec. per cm^2 per cm. head of water pressure.

Inspection of fabrics has shown that, in general, those whose permeabilities are higher than 150 units exhibit distinct holes in the texture, whilst those below this value rarely allow any direct transmission of light.

The change in the slope of the air permeability curve as indicated above is therefore attributed to the change in the mode of heat-transfer between the stream of air and the material, when the closer structure of the fabrics interferes with the free air-flow so that it changes its mode of percolation from direct to devious. Thus in consideration of heat-flow through air-permeable substances, the mutual influence of heat conditions and interchange of heat between the pervious fabric and the air stream is of importance.

Awbery⁽⁷⁾, in a discussion of heat-conduction through a granular substance dispersed in a medium of different conductivity, refers to the effect of heat-interchange between the two components. He states that this interchange would cause deviation from certain relatively simple laws, but gives no mathematical analysis of the effect. This is a parallel case to that discussed in the present paper.

§ 4. HEAT-LOSS FROM THE BARE CYLINDER

In the discussion given in the previous section, it has been assumed that convection and radiation play a considerable part in the transfer of heat between the hot inner and the cooler outer cylinders. In order to check the validity of this assumption, the heat losses from the bare cylinder by conduction, convection, and radiation have been calculated. The results are shown in the curve, figure 5, from which it will be seen that the convection and radiation effects are both larger than the conduction effect. There can be no very great change in the relative magnitude of these effects when a fabric is introduced, and hence the assumptions used in this paper may be taken as justifiable. A few words are appended on the calculations.

The conduction was calculated from the concentric-cylinder formulae, with a value of the thermal conductivity at 25°C . interpolated from those given by Kaye and Laby at 0°C . and 100°C . As the conduction is proportional to the temperature-difference θ a straight line may be drawn through the origin and any calculated point.

The only available information^(6,8) relating to convection from horizontal cylinders assumes that they are in an infinite volume of the cooling medium. In

the present case, however, the cylinder is enclosed concentrically in another nearly $2\frac{1}{2}$ times its diameter. With such an interspace there will be an approach to free-air conditions. The results of calculations made on that basis, however, will probably be too large, since the air approaching the cylinder will not have cooled down to the temperature of the outer tube. The heat-loss by convection from a horizontal cylinder has been found by many workers to be proportional to the $\frac{3}{4}$ -power of the temperature-excess above the surrounding air, and also to the area of the cylinder. The factor in the equation connecting the heat-loss with these quantities

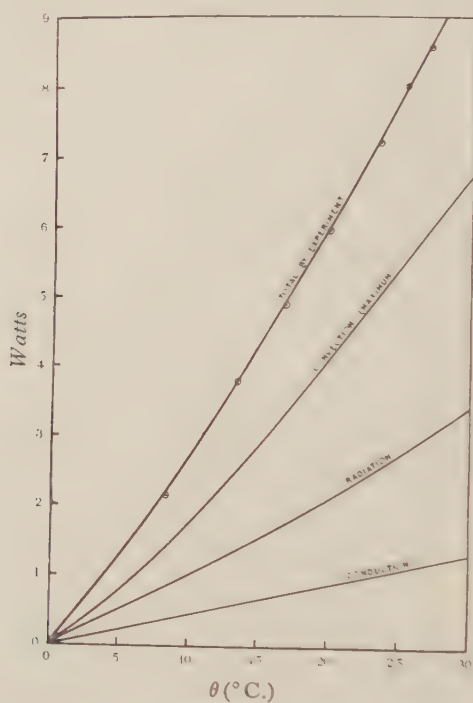


Figure 5.

depends on the diameter of the cylinder, and from a table given by Griffiths and Davis a curve has been plotted from which it was deduced that, for a cylinder-diameter of 5.50 cm., the above-mentioned factor has a value of 5.3×10^{-5} . Table 3 shows values calculated thus for the heat loss by convection for various temperature differences.

Table 3

Temperature-difference θ (°C.)	1	5	10	15	20	25	30
Heat-loss by convection (W.)	0.096	0.72	1.70	2.91	4.04	5.35	6.72

A similar table has already been given for the heat-loss by radiation, and these figures, multiplied by the emissivity which was found to be about 0.42, are plotted in figure 5 along with the conduction and convection values. Figure 5 also shows

the total heat-loss found by experiment as being rather less than the total of the three calculated components. This no doubt is largely due to the actual convection value being less than that calculated, owing to the assumptions regarding free-air conditions being hardly realized.

When the logarithm of the total heat-loss is plotted against temperature-difference it is found to give a straight line. The slope of this line would indicate that total heat-loss is proportional to $\theta^{1.20}$.

§ 5. ACKNOWLEDGMENT

The author wishes to thank the Research Control Committee of the Wool Industries Research Association for permission to publish this paper.

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DISCUSSION

Mr J. H. AWBERY. The paper gives a very interesting analysis, but I suggest that it may have read too much into the data. What is really known about the thermal insulating value as a function of permeability, after correction for radiation effects, is shown in the dotted curve of figure 2.

Consider now the apparatus set up without any fabric. There would be convection currents carrying heat from the inner to the outer cylinder. If a very open net of fabric is now inserted, it will make but little difference to the currents, and therefore to the heat-transfer. This is precisely what is found above permeabilities of about 170 units. Now reduce the permeability further, by using a finer mesh, and the currents are damped. They will carry less heat from one cylinder to the other, and this conclusion is quite independent of any knowledge of the detailed mechanism of heat-transfer at the fabric itself. The deduction is quite in accord with the curve, figure 2, and whilst it is independent of any considerations relative to heat-exchange at the fabric, it by no means proves that Mr Marsh's further conclusions are wrong. It only appears to render them superfluous.

AUTHOR's reply. Mr Awbery's explanation of the effect of air permeability on the thermal insulating value of a fabric is one that naturally occurs on the inspection

of the dotted curve in figure 2. It assumes, however, that fabrics which have a permeability above 150-170 units are open nets which have little or no effect on convection currents, while those below have a much closer structure which causes the currents to be damped. This arbitrary division is certainly not supported in practice. Many quite "solid" fabrics have a higher permeability than this, and to be termed a net a fabric would have to have a permeability of at least 1000 units. I should like to quote two experimental results to show that high-permeability fabrics can affect the convection currents and hence the heat flow. In the course of the smoke experiments described in the paper, a fabric with a permeability of about 800 units was used and this was found to have an effect on the convection currents which could be very easily observed. Further, I have shown that when a very open wire gauze cylinder (91 per cent. hole area) was inserted, the total heat flow was reduced by 3 per cent. The permeability of the gauze would be practically infinite. I regret therefore that I cannot agree that the explanation of the bend on the dotted curve of figure 2 is as simple as Mr Awbery suggests.

A COMPACT ELECTROMAGNET FOR GENERAL PURPOSES

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ABSTRACT. An electromagnet suitable for many branches of research work is described. Special attention is paid to the construction of the coils, which are cooled by the circulation of oil. Some tests and experiments with a magnetic potentiometer are described.

§ 1. INTRODUCTION

AN electromagnet was required for the investigation of the thermal and magnetic properties of ferromagnetic substances. Enquiry showed that few standardized magnets suitable for research purposes are made in England, whilst the present cost of many magnets of foreign manufacture is prohibitive. It was therefore decided to make the magnet here described, the construction being carried out in the Engineering Workshop at University College, London, by kind permission of Prof. E. G. Coker, F.R.S.

While it is not suggested that the design of the yoke and pole pieces of this magnet is particularly original, it is thought that some details of its manufacture will be of interest to other workers. At the same time it is felt that the effective arrangements made for the cooling of the magnetizing coils warrant some detailed description of them.

§ 2. DETAILS OF CONSTRUCTION

The constructional details of the magnet are given in figure 1. The yoke and pole-pieces, shown in figure 2, were of forged dead mild steel supplied by The English Steel Corporation, Sheffield. The steel was remarkably soft and easy to machine, having a Brinell hardness number of 110. The relevant magnetic data, obtained with a ring specimen of this steel, are given in table 1.

Table 1.

H (Gauss)	2.5	5.0	10.0	20.0	30.0
B	4900	9300	12300	13600	15500
H (Gauss)	40.0	50.0			
B	16200	16600			

The yoke was first planed all over and the holes for the pole pieces were bored. These two holes were finished at one setting with a broad tool acting practically as a scraper, the yoke being carried from the saddle of a lathe. The pole pieces, 4 in. in diameter, were then fitted to the bores by lapping.

The pole-tips are held to the pole pieces by central bolts which may be replaced by tubular bolts for certain experiments in magneto-optics. The use of separate pole-tips is not ideal from the point of view of prevention of magnetic leakage, but is necessary to allow of the provision of different shapes and special alloys for these tips.

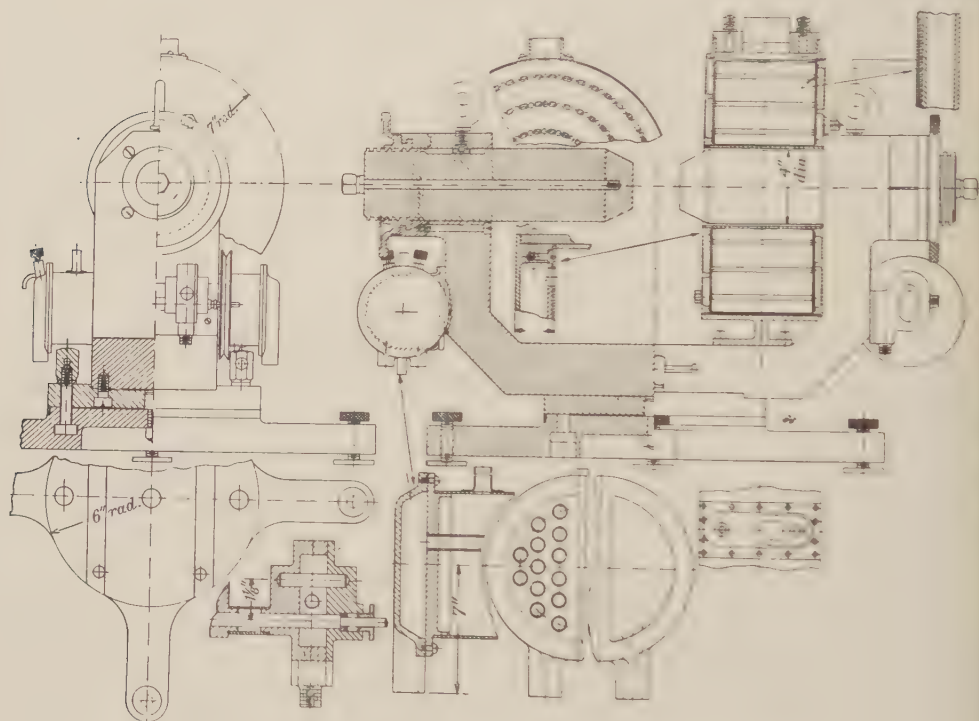


Figure 1. General arrangement and details of electromagnet.

The ideal shape for a fixed pole piece is, of course, a taper, but it was decided to make cylindrical pole pieces so that the width of the air gap might be more easily adjustable. A square thread, five to the inch, was cut on the outer end of each pole piece, engaging with a knurled brass handwheel held axially by a split cast-iron distance ring. The maximum axial adjustment on each pole piece is 2 in. A circular scale graduated in 200 parts on each handwheel allows the setting to be controlled to 0.001 in.

To prevent the pole pieces from turning, the lower ends of the eye-bolts, seen in figure 2, are machined to fit into holes in phosphor bronze keys engaging in suitable keyways in the pole pieces. These keys also limit the amount of the travel,

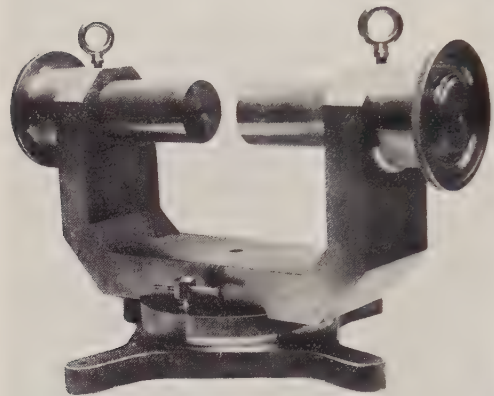


Fig. 2.

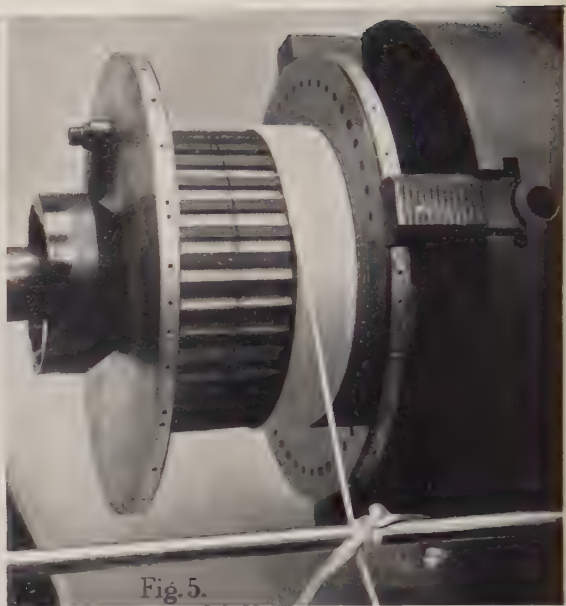


Fig. 5.

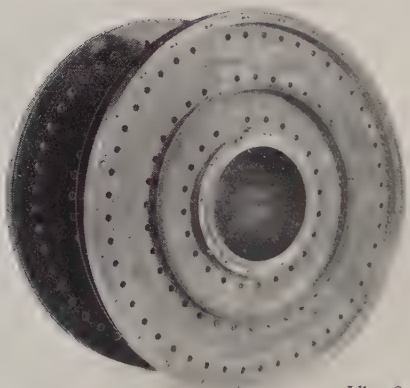


Fig. 3 a.

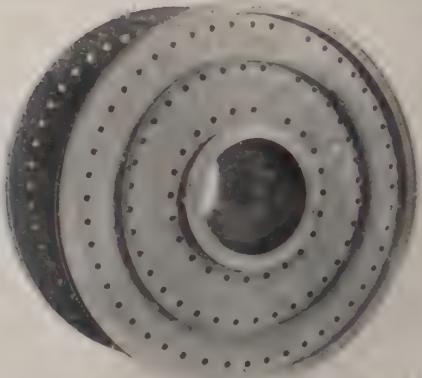


Fig. 3 b.



Fig. 4

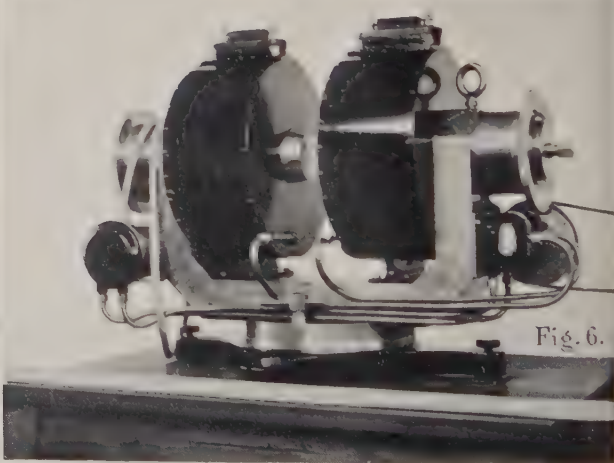


Fig. 6.

and the handwheels are kept comparatively small to prevent the keys or surfaces from being scored in case any foreign matter finds its way between the sliding surfaces. A half turn of an eye-bolt serves to lock the position of the corresponding pole piece. To preserve the micrometer adjustments of the handwheels from unnecessary wear, it is not intended that adjustment of the air gap should be made when the current is flowing.

Turntable. For certain experiments on crystals it is necessary to rotate the magnet through a measured angle about a vertical central axis. The yoke is therefore mounted on a cast-iron base-plate turning on a central pin in the manner of the slide-rest of a lathe. The amount of rotation is measured with a scale and vernier seen in figure 2 and made by Messrs Vinter, Dyer's Buildings, Holborn, W.C. 1.

The upper and lower plates of the turntable bed over the outermost $\frac{3}{8}$ in. only, in order to reduce friction to a minimum, with the result that the whole magnetic system can be rotated comfortably by hand.

Magnetizing-coils; formers. Each coil former consists essentially of a central brass tube carrying two annular side cheeks, or boxes of brass, and enclosed by a copper wrapper plate so as to form a drum. This is oil-tight except at the uppermost part, where a well is formed, and through which the two electrical leads are taken.

In the actual design, to save time in pattern-making, the cheeks of the formers were built up, but experience showed that the outer sections of the cheeks could be more easily constructed in the form of castings. This improved design is therefore incorporated in figure 1. The whole of the coil former was built up and fitted, and the appropriate portions were tinned before any wire was wound on. Thus the only joints to be made after the wire was in place were the circumferential joints between the wrapper plate and the outer rings of the side cheeks.

On reference to figures 3 *a* and 3 *b* which, of course, are pictures of the built-up former, it is seen that in the inner plate of each cheek there are three concentric rings of holes and a spacing ring. The latter prevents the radial flow of oil as described below, and as seen in figure 3 *a*, to which reference is now made.

The oil is led in by a duct, seen in figure 4, to the innermost set of holes from which it flows axially, i.e. parallel to the lines of magnetic induction, to the opposite cheek through passages seen in figure 5 and described below. It now flows radially to the middle set of holes seen in figure 3 *b*, and, further radial flow being prevented by the spacing ring, the oil now passes through these holes, axially crossing the windings to the middle set of holes in the first cheek, shown in figure 3 *a*. It now flows radially to the outermost set of holes, and thence again axially across the windings to reach the outer annulus of figure 3 *b*. From here it flows through a duct, not visible in figure 4, to the cooling system.

The insulation of the former consists on one side of a single disc of Paxolin $\frac{1}{16}$ in. thick, while on the opposite side three discs are used. The outer pair here are each $\frac{1}{16}$ in. thick, while the central disc is only $\frac{1}{64}$ in. thick, being cut away radially for a width of $1\frac{1}{4}$ in., to admit the passage of a strip of copper, $\frac{1}{4}$ in. thick, which acts as the electrical connexion or ingoing lead to the innermost layer of the

coil. The insulation over the central brass tube consists of a cylinder of Paxolin $\frac{3}{8}$ in. thick, split axially as it had to be put in position in halves.

In the course of the manufacture, the inner brass plate of one of the side cheeks was marked off and the sets of holes, each $\frac{1}{4}$ in. in diameter, were drilled at one setting in all four brass plates and the eight Paxolin discs. As may be seen from figures 3*a* or 3*b*, no holes are drilled opposite the ingoing lead. The holes in the side cheeks and the corresponding insulation are kept in line by a tubular bushing in one of the holes.

The formers have an internal axial length of $5\frac{3}{8}$ in. and a radial dimension of $4\frac{1}{2}$ in.

Winding of the coils. It was decided to use double-cotton-covered copper wire of no. 12 s.w.g., so that with a current of 20 A. a total of about 61,000 ampère-turns is available. The winding was put on in uniform layers until it lay flush with the inner edges of any one of the three sets of holes. Axial bars of red fibre, $\frac{3}{8}$ in. \times $\frac{3}{16}$ in. in section, were then placed between corresponding pairs of holes in the two cheeks of the former in order to leave an axial set of passages for the oil. The bars were held temporarily in position by thread as shown in figure 5, and more turns were wound on.

When the final layer was in place a turn of red fibre sheet $\frac{1}{16}$ in. thick and $5\frac{3}{8}$ in. wide was wrapped around the completed coil in order to protect the winding from heat during the soldering of the wrapper plate. The final layer of wire failed to extend right across the former, so as to ensure that the last turn was almost vertically below the outgoing terminal to the coil. Both terminals were mounted in a hollow block of teak fitted in the brass well, which served to accommodate to some extent the expansion of the oil when the latter became heated.

Each coil has its own gear wheel, pump and section of the cooler, as shown in figure 1, the pumps being driven by an external motor. The relief taps shown at the top of the cooler allow for the release of any air collected in the cooling system, and for the release of oil, in case the level rises too high in the well at the top of the coil.

Transformer oil, B30, supplied by the Vacuum Oil Co., was used, but before each former was filled, dry air was drawn through the path normally taken by the oil, a heating current of some 5 A. being maintained through the coils to remove as far as possible any moisture from the cotton insulation; this precaution is most essential. Each former contained approximately one gallon of oil, and this was forced through the cooling system about once per minute.

Each coil contains approximately 125 lb. of wire, and the total weight of the magnet and its attachments is about 600 lb. The complete magnet is shown in figure 6.

§ 3. PERFORMANCE

The magnet can be operated at 20 A. for considerable periods. Some idea of its performance may be gained from the following details. In table 2 are given the values of the axial field mid-way between two flat pole-tips, tapered from 9.7 to

8.4 cm. in diameter, with a gap of 1.5 cm. between them and the stated currents supplied to the coils.

Table 2

Current (A.)	1.0	3.5	5.0	8.0	15.0	23.0
Field (G.)	2776	9030	11100	13200	15250	16710

With the same pole-tips and a constant current of 20 A., the field mid-way between them varied with the width of the gap as set forth in table 3.

Table 3

Gap (cm.)	1.035	1.81	4.005	7.015	10.30
Field (G.)	19110	15060	9750	6620	4836

With the same pole-tips and a constant current of 20 A. the variation of the axial field with position in the gap was investigated. Some results for stated gaps are given in table 4.

Table 4

Gap (cm.)	Axial field at pole surface	Axial field mid-way between poles
4	9835	9780
6	7890	7500
8	6890	5990
10	6400	4925

With conical pole-tips, having an included angle of 120° and base 4 in. in diameter, whose ends were flat over an area 1 cm. in diameter, a mean field of 32,500 G. was obtained over an area of 12.6 mm² when the gap was 0.5 cm. and a current of 20 A. was employed.

Flux-losses. To obtain a measure of the flux-losses, a single loop of insulated wire was wound tightly on the yoke, or on the pole piece, and was connected to a flux-meter. The reading of the latter was recorded when the magnet current was reduced from 20 A. to zero, allowance being made for residual magnetism. With flat, non-tapered, pole-tips 0.85 cm. apart the data of table 5 were obtained when the loop was placed along the pole piece at the stated distances from the gap, i.e. from the pole-face.

Table 5

Position of loop (cm.)	0.05	1.0	2.0	3.0	4.0	5.05
Total flux through loop (Maxwells × 10 ⁶)	1.406	1.525	1.604	1.653	1.693	1.757

When the loop was in position at 5.05 cm. it was as close as possible to the coil-face. Consequently, in these tests, the ratio of the flux crossing the gap to the flux crossing the iron just at the exit from the coils was 0.80. The total flux over each portion of the yoke inclined at 45° to the horizontal was 2.18 × 10⁶ Maxwells. When the data of table 5 are plotted the graph obtained shows a point of inflexion

for the loop in the position 2.5 cm. from the pole-face. This distance is practically equal to the distance at which the minimum value of H_x , shown in figures 7 and 8, occurs.

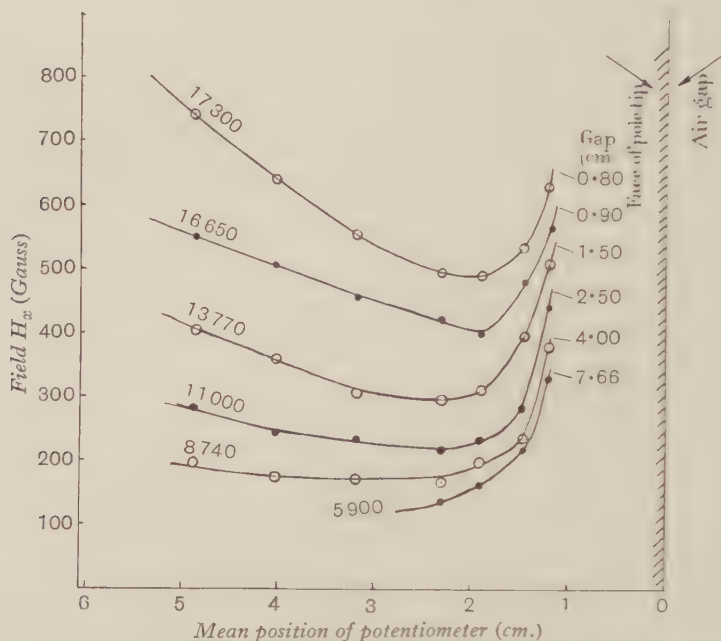


Figure 7. Variation of H_x with position on the pole piece. Plane pole-tips. Gap adjusted.

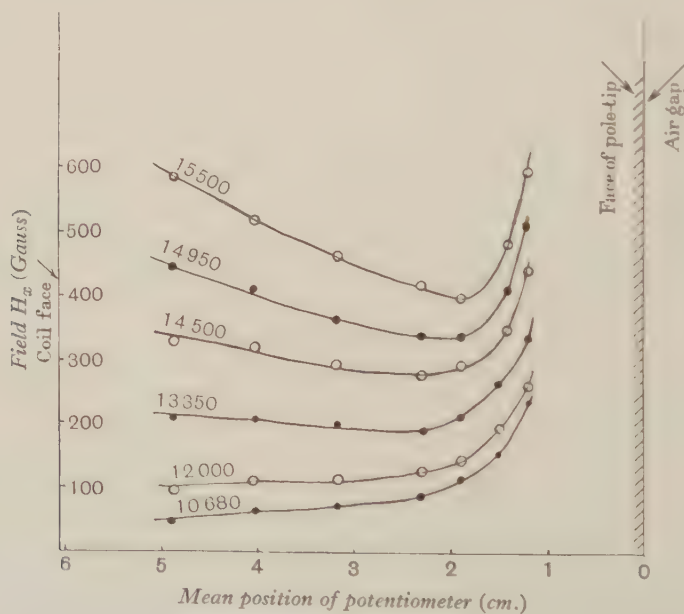


Figure 8. Variation of H_x with position on pole piece. Plane pole-tips. Gap 1.18 cm. Current adjusted.

Tests with a magnetic potentiometer. Some further interesting information was obtained by the use of a simple form of magnetic potentiometer*. This was made by taking an ebonite rod 5 mm. in diameter, with a small hole running axially through it, and winding upon it a solenoid of fine enamelled wire for a length of about 5 cm. To protect the wire a layer of cotton thread was wound upon the solenoid, and the ends of the solenoid and the cotton were secured with touches of hard wax. The ebonite rod was now placed in hot water and the portion on which the solenoid was wound was bent into a semicircular arch around a reel. On removal from the water, the ends of the ebonite rod were pushed through two holes in a small ebonite plate, so that the ends of the semicircle were flush with the lower surface of the plate. The semicircular piece, or half-ring, of ebonite was then cemented in position.

The wire of those portions of the solenoid which projected beyond the lower surface of the plate was unwound until the lowest turn on each side was flush with this surface. Any surplus ebonite rod was carefully filed away. The wire from one end of the solenoid was then threaded through the axial hole in the ebonite, so that the ends of the solenoid were now close together. These ends were cemented into a groove cut in the side of the plate, and stouter wires, to serve as leads, were soldered to them.

It was found advisable to remove as much as possible of the portions of the plate outside the semicircle, so that one could clearly gauge the outer limits of the solenoid. Thus, finally, a magnetic potentiometer consisting of a uniformly wound coil of semicircular shape, with about 450 turns each about 20 mm² in area, was obtained, the external diameter of the ring being 2.4 cm.

The ends of the coil were connected to a sensitive moving-coil ballistic galvanometer of long period, and the potentiometer was used to measure the field actually acting upon various portions of the yoke and pole pieces. The potentiometer was placed on a part of the magnet where the field was to be measured, with the lower surface of the ebonite plate immediately in contact with the iron, and with the axis of the solenoid in the same plane as the direction of the field on the iron. On sudden removal of the potentiometer to a field-free region a deflection of the galvanometer resulted. This deflection was a measure of the difference in magnetic potential between the two points in the iron immediately below the centres of the lowest turns on the solenoid, and, therefore, of the mean field in the region between these points; in what follows this field is denoted by H_x . The potentiometer was calibrated by removing it from a uniform magnetic field, such as existed between flat pole-tips when a small current flowed through the magnet coils, the field being measured by a flux-meter.

H_x

With the pole-tips 8.4 cm. in diameter, a gap of 1.043 cm. and a current of 20 A. through the magnet coils, giving a field of 18,800 G. in the gap, the value of H_x over the horizontal portions of the pole pieces at a mean distance of 4.5 cm. from the edge of the gap was 356 G. On the middle of the sloping portion of this pole-tip, the value of H_x parallel to the sloping metal face was 750 G., while on the middle

* Cf. W. Wolman, *Arch. Elektrot.* **19**, 385 (1928). W. Rogowski and W. Steinhaus, *Ibid.* **1**, 141 (1912). A. P. Chattock, *Phil. Mag.* **24**, 94 (1887).

of the vertical portion of the yoke it was 130, on the inclined portion of the yoke 69, and on the middle of the base about 5 G.

With flat pole-tips 10 cm. in diameter, a gap of 0.93 cm. and a current of 20 A. giving a field of 15,000 G. in the gap, the base value of H_x was about 3 G. With these tips we also obtained the sets of curves shown in figures 7 and 8. In figure 7 are plotted the values of H_x for various mean positions of the magnetic potentiometer along the horizontal portions of one of the pole pieces, the current being usually maintained at 20 A. and measurements made with a series of gaps. The vertical line on the right of the figure represents the edge of the pole-tip, and the line on the left represents the face of the coil. The field in the gap is recorded for each curve.

It was impossible, without using a much smaller potentiometer, to obtain values of H_x for regions nearer the gap. Indeed, in using the instrument close to a gap some skill in effecting the sudden removal was necessary, or the potentiometer would have been plunged into the huge field around the gap and false readings would have been obtained. Again, in fields which were very non-uniform the inevitable slight lack of uniformity in the winding of the potentiometer produced marked effects; the deflections on opposite sides of zero showed considerable differences in some cases. Mean values were always taken. Therefore it is not suggested that these curves are highly accurate. It is considered, however, that they give an interesting picture of the way in which the field actually acting upon the pole pieces varies, and that the results should be of assistance in choosing alloys for special pole-tips.

Figure 8 shows how the field on the pole piece varied with position when the gap was kept fixed and the current in the magnet coils adjusted to give different values of the field in the gap. In both figures 7 and 8 the curves for high fields in the gap show very pronounced minima. The minima disappear completely when the fields in the gap are small. The curves suggest that with a magnet such as is described here, pole-tips of alloys of high permeability should be at least 3 cm. thick for the production of most intense fields.

These measurements were checked with another potentiometer of slightly larger span, wound with two layers of wire, in all about 900 turns. The differences between readings on opposite sides of zero were much more pronounced than for the first potentiometer, but, on plotting the mean values as before, we obtained identical curves.

§ 4. ACKNOWLEDGMENTS

The authors have already recorded their indebtedness to Prof. Coker; it remains for them to thank Mr T. Gurman of the Engineering Workshop Staff for his excellent workmanship in the construction of this magnet, and also Mr L. Walden of the Physics Departmental Staff who furnished the photographs reproduced above.

One of the authors (L. F. B.), also desires to record his deep appreciation of a grant from the Government Grants Committee of the Royal Society, which covered the entire cost of the magnet, and of the facilities in the Physics Department afforded him by Prof. E. N. da C. Andrade.

DISCUSSION

Mr R. S. WHIPPLE said that the method of cooling seemed to be efficient and might serve for other instruments also. He asked what magnetic flux was attainable?

Mr J. GUILD enquired how the magnetic flux compared with that of an electromagnet of the du Bois type for the same size of gap?

Prof. A. O. RANKINE asked what was the saving in cost of production effected by the authors?

Dr L. F. BATES. The maximum flux so far obtained is 32,500 Gauss with a gap of 0.5 cm. between conical pole-tips having an included angle of 120° and base 4 in. in diameter, the ends being flat over an area 1 cm. in diameter; obviously, this flux could have been greatly increased by reducing the width of the air gap. The values of the magnetic flux compare very favourably with those recorded for a large magnet of the du Bois type. An interesting table of the comparative performances of several different types of electromagnet will be found in the latest edition of Müller-Pouillet's *Lehrbuch der Physik*, 4, 418. We have not attempted to establish records, but have made a magnet which we consider particularly suitable for use in certain researches that one of us has in hand, and very suitable for general purposes. The approximate cost of the magnet was £100, of which £50 was expended on labour; the saving was therefore estimated to be about £150, overhead charges being, of course, very small.

THE SPURIOUS RING EXHIBITED BY FLUORESCENT SCREENS

By J. V. HUGHES, A.R.C.S., B.Sc.

Received December 21, 1932. Read, with demonstration, February 17, 1933.

ABSTRACT. The spurious ring exhibited by fluorescent screens used for the observation of electrons is explained as being due to total internal reflection of the light at the upper surface of the glass block of the screen. The theoretical intensity-distribution is deduced and is compared with experiment by the use of a specially designed photometer. The experimental results agree well with the theory.

FLUORESCENT screens used for the observation of electrons always show a ring round the bombarded spot, known as "the spurious ring." In general this ring has a very sharp inner edge, and gradually fades away on the outer side. It is usually explained as being due to light from the central spot which has suffered internal reflection in the glass and has returned to the material of the screen, whence it is scattered by the particles in all directions.

According to this theory, the inner edge of the spurious ring corresponds to the ray which is just totally internally reflected at the upper surface of the glass.

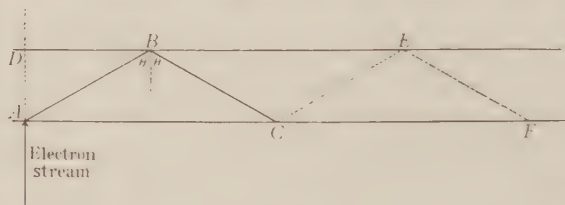


Figure 1.

According to this theory, light from the point of bombardment A , figure 1, is totally internally reflected at B , and strikes the material of the screen again at C , whence it is scattered in all directions. Thus, no matter in what direction the screen be viewed, the point C is observed bright. Rotation of the diagram about AD produces the spurious ring of radius AC .

Some of the light scattered from C is again internally reflected, as at either B or E in the diagram: in fact every point (such as C) on the first spurious ring gives rise to another complete ring round that point. The envelope of these secondary rings is a circle round AD , of radius AF . This is a possible explanation of the second spurious ring which is sometimes seen, but the sharpness of the second ring, when it is observed, would lead one to think that some of the light is not scattered at C but regularly reflected up to E , whence it is reflected again to F ,

where it is scattered by the particles of the screen. This explanation of the second spurious ring leads one to expect a sharper second ring than one would expect from the first explanation, and makes it easier to account for the third spurious ring which is sometimes observed.

If θ be the angle of reflection, D the diameter of the spurious ring, and t the thickness of the glass, then $AC = \frac{1}{2}D$ and clearly $\tan \theta = D/4t$.

But for the critical angle of total internal reflection $\sin \theta = 1/\mu$ where μ is the refractive index of the glass.

Thus
$$\mu = \operatorname{cosec} \tan^{-1} \frac{D}{4t} = \frac{4t}{D} \sqrt{\left(1 + \frac{D^2}{16t^2}\right)}.$$

Thus a measurement of D and t gives us a value of μ . These measurements were carried out for four screens, the thicknesses being measured with a micrometer screw gauge and the diameters of the rings by means of a pair of dividers.

To check the theory, the refractive indices of the glass blocks used for the screens were obtained in the usual way, by focusing a microscope first on the material of the screen, seen through the glass, and then on the upper surface of the glass, and thus getting the apparent thickness T of each of the screens in turn. The true thickness t being already known, the refractive index was obtained from the relation

$$\mu = \frac{t}{T}.$$

In all cases the refractive indices of any one screen, as obtained by these two separate methods, agreed to within the limits of experimental error.

The results are given in the table. Screen A was willemite floated on in water-glass solution. Screens B, C, D, were zinc blendes spread on with a solution of Canada balsam in xylol.

	Screen A	Screen B	Screen C	Screen D
Thickness t (cm.)	0.84	0.78	0.765	0.77
Ring-diameter D (cm.)	3.1	2.7	2.6	2.6
μ from these	1.47	1.53	1.54	1.55
μ from optical method	1.50	1.51	1.52	1.51

Two points of interest remain. One is that the light, after its one internal reflection, is able to reach the material of the screens (which is of course outside the glass block) and be scattered by it. The scattering material must therefore be in good optical contact with this second (lower) surface. The explanation of this lies in the fact that the material of the screen is held in position by an adhesive, either waterglass or Canada balsam, in which the particles of the screen are embedded. For the purposes of the light, these adhesives constitute practically an extension of the glass (as both materials have refractive indices of about the same value as that of glass) so that the particles of the material of the screen are optically in the glass, and hence can be acted on by the light.

The second point is to see whether theory can predict the approximate intensity-distribution, and particularly the sharp inner edge of the ring. In carrying out this

investigation three assumptions are made: (a) The particles are sufficiently separated from one another as not to interfere with each other. Thus a cosine law of distribution of energy with angle is not involved. (b) The particles emit light in all directions. This is rather doubtful, but allowance for the deviation would be rather difficult, and the error introduced by making this assumption will probably have little effect on the shape of the intensity curve. (c) The light re-radiated from the ring of course contributes light to other parts of the screen. This secondary effect is taken to be negligible.

ϕ
 n We apply Fresnel's formulae, which for reflection at an angle ϕ in a material of refractive index n can be written:

$$A_3 = A_1 \frac{n^2 \cos \phi - \sqrt{(n^2 - \sin^2 \phi)}}{n^2 \cos \phi + \sqrt{(n^2 - \sin^2 \phi)}}$$

$$B_3 = B_1 \frac{\cos \phi - \sqrt{(n^2 - \sin^2 \phi)}}{\cos \phi + \sqrt{(n^2 - \sin^2 \phi)}}$$

B_3, A_1, B_1
 I where A_3, B_3 , are reflected amplitudes due to amplitudes A_1, B_1 , plane polarized perpendicular to and in the plane of incidence respectively. If I be the reflected intensity for unpolarized light

$$I = A_1^2 \left\{ \left(\frac{n^2 \cos \phi - \sqrt{(n^2 - \sin^2 \phi)}}{n^2 \cos \phi + \sqrt{(n^2 - \sin^2 \phi)}} \right)^2 + \left(\frac{\cos \phi - \sqrt{(n^2 - \sin^2 \phi)}}{\cos \phi + \sqrt{(n^2 - \sin^2 \phi)}} \right)^2 \right\}.$$

θ Here n is the refractive index of the glass in the direction of travel of the light, so that if θ be the critical angle of total internal reflection, $\sin \theta = n$, and for values of ϕ greater than θ we merely neglect the factors under the root signs and put,

$$I = 2A_1^2.$$

From figure 2,

$$\tan \phi = r/2t,$$

$$\text{whence } I = A_1^2 \left\{ \left(\frac{2n^2t - \sqrt{n^2(r^2 + 4t^2) - r^2}}{2n^2t + \sqrt{n^2(r^2 + 4t^2) - r^2}} \right)^2 + \left(\frac{2t - \sqrt{n^2(r^2 + 4t^2) - r^2}}{2t + \sqrt{n^2(r^2 + 4t^2) - r^2}} \right)^2 \right\} \\ = A_1^2 f(r) \text{ say.}$$

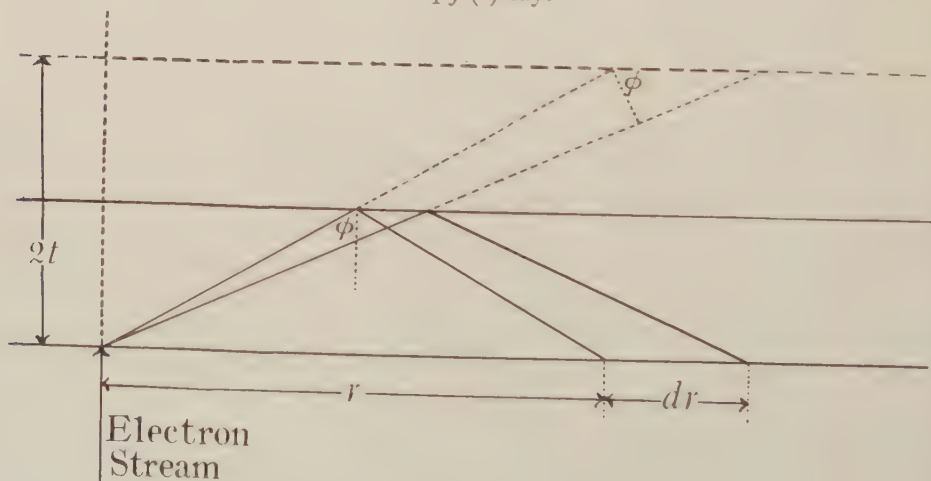


Figure 2.

The solid angle subtended by annulus of width dr at radius r is

$$\frac{2\pi r \cdot dr \cdot \cos \phi}{AB^2}, \text{ which } = \frac{\pi 2r \cdot dr \cdot \cos \phi}{t^2 \sec^2 \phi} \\ = \frac{2\pi r \cdot dr \cdot \cos^3 \phi}{t^2}.$$

The area of the annulus is $2\pi r \cdot dr$.

Thus if X per unit solid angle be emitted equally in all directions from A per second, the amount reaching this area per second, is

$$X \cdot 2\pi r \cdot dr \cdot \cos^3 \phi \cdot f(r)/t^2,$$

where $f(r)$ is the function defined above to allow for the reflection. Thus energy per unit area per second, falling at radius r , is

$$X \cdot \cos^3 \phi \cdot f(r)/t^2 \text{ which } = \frac{X}{t^2} \left(\frac{4t^2}{r^2 + 4t^2} \right)^{\frac{3}{2}} f(r).$$

Substituting for $f(r)$ we get as a final expression that the intensity at radius r is proportional to

$$\left(\frac{1}{r^2 + 4t^2} \right)^{\frac{3}{2}} \times \left\{ \left(\frac{2n^2t - \sqrt{\{n^2(r^2 - 4t^2) - r^2\}}}{2n^2t + \sqrt{\{n^2(r^2 + 4t^2) - r^2\}}} \right)^2 + \left(\frac{2t - \sqrt{\{n^2(r^2 + 4t^2) - r^2\}}}{2t + \sqrt{\{n^2(r^2 + 4t^2) - r^2\}}} \right)^2 \right\}.$$

Putting $n = 1/1.5 = 0.667$ and $t = 0.77$ cm., we get the theoretical curve shown in figure 3. The maximum, where $r^2 = n^2(r^2 + 4t^2)$, occurs at $r = 1.378$ cm., or a spurious-ring diameter of 2.76 cm. When r exceeds this value, i.e. when $r^2 > n^2(r^2 + 4t^2)$, we take the intensity as being proportional to $2/(r^2 + 4t^2)^{\frac{3}{2}}$.

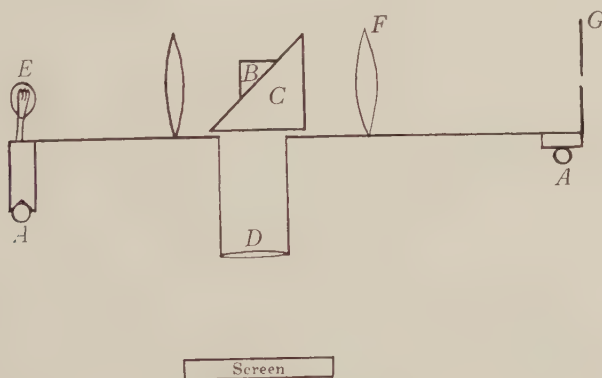


Figure 3. The photometer.

The inner edge of the curve rises extraordinarily steeply, and the fall away on the other side is gradual, in qualitative agreement with the observed effect. To test the agreement quantitatively, a photometer was constructed so that it could slide on rails A , figure 4, parallel to the edge of the screen when the latter was in position on the apparatus. I am indebted to Mr Warburton for the design of this photometer and for the interpretation of the results obtained by its use.

A photometer cube was used and is shown in figure 3. Light from the fluorescent screen, rendered parallel by *D*, was totally internally reflected in *C*. Light from a small bulb *E*, after traversing a suitable colour-filter, was collimated and passed directly through *C* and *B*. The field was observed with a telescope *FG*. By suitable placing of the pinhole eyepiece *G* a bipartite field could be obtained. The intensity of illumination from the bulb *E* could be reduced by means of an absorption wedge whose position could be read on a scale.

The readings of the wedge for equal illumination of the two halves of the field were obtained for various settings of the photometer on its rails (read on another scale), i.e. for various distances from the centre spot. The law of the absorption wedge being known, these readings could be converted into intensities.

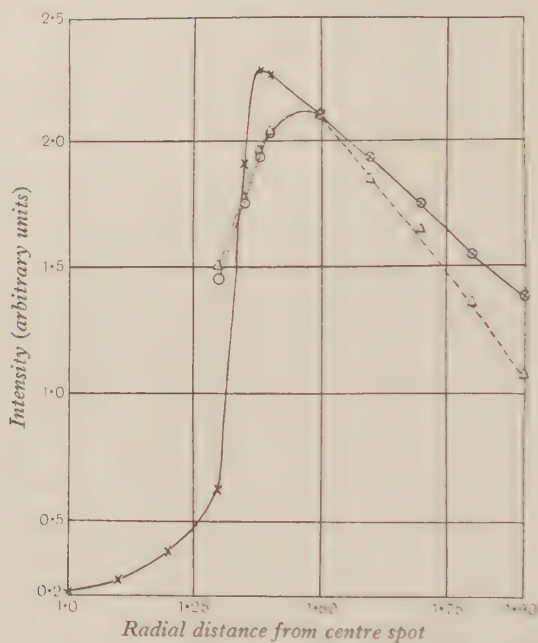


Figure 4.

- × — × Theoretical curve
- ---- ○ Curve corrected for finite scanning area
- △ ---- △ Experimental curve

One correction has to be made. To obtain sufficient intensity, a large eyepiece-aperture was needed. Thus the intensity observed was not the intensity at a point but the mean over an area. The correction on this account is most readily effected by modifying the theoretical curve to allow for a finite scanning area, taken as circular. This correction also was carried out by Mr Warburton, and the theoretical curve, the modified curve, and the observed curve are all plotted on one sheet of graph paper in figure 4.

The experimental values, which are only relative, are adjusted to make the maxima coincide. The inner edge agrees well but the outer edge falls away more rapidly than the simple theory would indicate. Evidently, one of the assumptions made in the theory does not hold quite accurately.

Summing up, one can say that the theory of the rings here advanced—that they are due to reflection of the light emitted from the centre spot by the upper surface of the glass—seems adequate to account both for the size of and the intensity-distribution in the rings.

DISCUSSION

Mr T. B. VINYCOMB. At the Physical Society's Exhibition a year ago one of the firms of apparatus-makers exhibited an arrangement for measuring the index of refraction of a slab of glass by means of this phenomenon.

Prof. G. P. THOMSON. I think the author has done a neat piece of work. This effect is very striking and it is curious that it has not been more noticed in the literature. It might easily lead an experimenter who was unaware of it to false conclusions, but the method of photographic recording by the actual impact of the rays obviates the danger.

Dr W. D. WRIGHT. Surely any doubt as to the cause of the spurious ring could have been removed by wetting the outer surface of the cathode tube. This would have increased the critical angle and hence the diameter of the ring.

Mr E. F. FINCHAM. The phenomenon described in this paper appears to be the same as the "halation ring" which is well known in photography. If a small source of light is photographed upon an unbacked plate and sufficient exposure is given, the image of the source is found to be surrounded by a flare and around this is seen a dark zone surrounded again by a bright ring produced in the same way as the effect described in this paper.

Mr J. GUILD. The author has effectively traced to its source a phenomenon which appears to have been a mystery as well as an annoyance to workers in some branches of physics. I cannot help feeling, however, that a special *ad hoc* investigation of the cause of this halo ought to have been quite uncalled for. It is one of these simple and familiar phenomena, many of which are seen "by the wayside" in every laboratory, of which the explanations are so immediately obvious as to elicit no comment. The halo is not, as one might gather from this paper, specially associated with fluorescent screens. It is observed in every case in which there is a local concentration of light on a transparent plate with one matt and one smooth surface, as, for example, when a small image is focused on a sheet of ground glass or of flashed opal. The phenomenon is described, and its contribution to the behaviour of pot opal glasses investigated, in a paper by J. S. Preston*.

In so far as the halo is a nuisance to those experimenting in electron-diffraction it should be possible to work with an opaque screen and arrange the optical system so that the screen is viewed or photographed from the front.

* *Proc. International Illumination Congress*, 1, 373 (1931).

Mr T. SMITH suggested that the spurious ring could be easily distinguished if a non-parallel plate were used, non-concentric rings being then rejected.

Prof. A. F. POLLARD. If these spurious rings are troublesome, may I suggest a construction of the fluorescent screen which would eliminate them. Make a collodion, or better a cellulose acetate, emulsion of the fluorescent crystals and cast a film on plate glass. Strip the film in the usual way by lowering the plate into water and pick up the film on the shellaced edge of a brass frame. As the film loses moisture it will contract and become taut. Paste a paper mask on the glass plate of the screen so as to leave a clear space of clean glass in the centre, and lower the film upon the paper edging which has been previously painted with shellac varnish. When the shellac is dry cut the brass frame free. The screen now consists of a thin collodion fluorescent film separated a few hundredths of an inch from the supporting glass plate, and internal reflections will not affect it. Diagonal airways must of course be cut in the paper edging to allow the escape of the air between film and plate during evacuation.

AUTHOR'S reply. In reply to Mr Vinycomb: As far as I am aware, the theoretical intensity-distribution in the ring had not previously been deduced or compared with experiment. In reply to Prof. Thomson: It would be convenient if we could photograph the images on the screen from the outside of the apparatus, as then the vacuum need not be disturbed in plate-changing. The spurious ring renders this method of recording undesirable. In reply to Dr Wright: Unless a layer of water of thickness comparable with that of the block of glass were used, the increase in the diameter of the spurious ring would not be within the limits of experimental observation. In reply to Mr Fincham: The "halation ring" in photography appears to be due to the same cause as the ring here described. In reply to Mr Guild: The explanation of the phenomenon has usually been taken for granted, but it was considered desirable to verify our suspicions on the subject and demonstrate conclusively that our assumed explanation was correct. Using an opaque screen and viewing from the front involves viewing from that side of the screen which is in the vacuum. This involves viewing through an observation window, and at an angle with the normal to the screen, so as not to interfere with the beam. Whilst this suffices for settings preliminary to photographing, it is sometimes desirable to take measurements on the screen, which would be difficult with a screen viewed in this way. Mr Smith's suggested arrangement can be shown also to distort diffracted rings seen on the screen, and hence to render measurements on the screen unreliable. In reply to Prof. Pollard: The screen suggested suffers from the drawback that despite the airways through the paper edging, the collodion film would probably be sucked off the glass if the vacuum were turned on suddenly. As a matter of fact, in practice the spurious ring is not as troublesome as one would expect it to be from its intensity. This is probably due to the greater sharpness of the true diffracted rings and to the fact that the eye is more sensitive to sudden changes of intensity than to the actual magnitude of the intensity.

THE DIRECT RECORDING OF RELATIVE INTENSITIES BY MEANS OF A MICROPHOTOMETER

BY N. THOMPSON, B.Sc., Physics Department, The University, Sheffield

*Communicated by Prof. S. R. Milner, F.R.S., December 3, 1932, and in revised form
January 25, 1933. Read February 17, 1933*

ABSTRACT. A description is given of an addition to the usual form of recording microphotometer, which gives a record on which ordinates are linearly proportional to light-intensities. Results are quoted to show that the performance of the instrument compares favourably with that of the unmodified form.

IT is well known that with a recording microphotometer of the usual type a curve is obtained whose ordinates are not linearly proportional to the intensities of the light which caused the blackening of the photographic plate. By measuring the record a series of photographic densities are found, and a calibration curve, obtained by photometering a set of density marks, must be used to convert these into relative intensities. For the most accurate work, where areal rather than peak intensities are required, this involves measuring a number of ordinates for each line in the spectrum, converting to relative intensities, re-drawing the curve, and then obtaining its area with a planimeter. This is seen to entail an immense amount of work, which in the past has usually been prohibitive. As a result, peak intensities have been used even for comparing lines whose (unresolved) fine structure is different, and erroneous results have ensued. As a further result, practically no reliable information has been obtained on the relative intensities of bands in a band spectrum, where the peak intensity is of very doubtful significance. The apparatus to be described, when used in conjunction with such a microphotometer, gives a record which shows the relative intensities directly, and thus enables determinations of areal intensities to be made merely by using a planimeter. In addition, if a large number of lines are to be compared for which the same (intensity, density) calibration curve is applicable, the apparatus will greatly expedite the process, even when the peak intensities are used.

As a preliminary, we will define a quantity which is a measure of the amount of blackening of the photographic plate, and one which, for our purposes, is more convenient than the density d which is usually used. This density is defined as $\log (a_0/a)$, where a is the deflection of the recording galvanometer and a_0 is the maximum value of a , obtained when the light is traversing an unexposed part of the photographic plate. The quantity we will use is defined as $(a_0 - a)/a_0$, the symbols having the same meaning as before, and this we will refer to as the *blackening* b . The (intensity, blackening) calibration curve is very similar in shape

d
 a, a_0

b

to the (intensity, density) curve, with the important difference that it is asymptotic to the line $b = 1$, instead of going off to infinity; see the plate, (c).

Suppose now we have an opaque screen, in which is cut a fine slit in the shape of an (intensity, blackening) calibration curve, and suppose the spot of light, after reflection from the mirror of the recording galvanometer, falls on this in the form of a fine line parallel to the axis of intensity (I). This spot is made to lie along the line $b = 1$ when undeflected, and along the line $b = 0$ when in the position of maximum deflection. Then at any instant its intercept on the axis of b will give the blackening of that part of the negative which is at that moment being photometered. Moreover, that portion of the light which emerges through the screen in the form of a small spot will have suffered a displacement, the component of which parallel to the axis of I will be proportional to the corresponding intensity. We therefore only require an optical system which will reproduce this deflection parallel to I on the revolving drum, while at the same time taking no account of the motion parallel to b , in order to produce a record on which the ordinates are proportional to intensities.

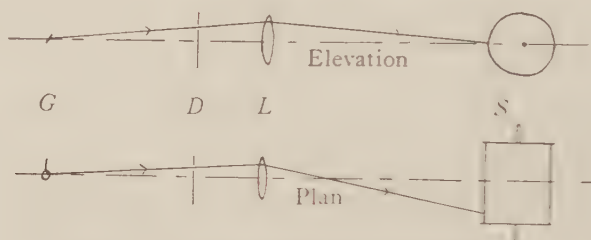


Figure 1. The optical system of the instrument.

Such a system is shown diagrammatically in figure 1. G is the galvanometer mirror, S the recording drum of the microphotometer, and D the pierced template described above. L is a special lens which is equivalent in its optical properties to two cylindrical lenses of different focal lengths, cemented together with the axes of the cylinders at right angles. As the axis of the recording drum which was to be used was horizontal, there was inserted between G and D an arrangement of two right-angled prisms cemented together (R , figure 2) which had the effect of making the galvanometer appear to deflect in a vertical plane. If the drum be made to rotate about a vertical axis, this arrangement can, of course, be dispensed with: it is shown in figure 2, but not figure 1. The template is placed with the axis of b vertical. In the vertical plane, L forms an image of G on S , so that however much the galvanometer deflects, the spot of light is always on the same horizontal line on S . In the horizontal plane, L has a shorter focal length, and D and S are conjugate foci, so that the movement of the spot on S is proportional to the horizontal component of its motion on D , as required.

In order to obtain an image of a slit on D , formed by a beam of light which has been reflected from the galvanometer mirror, the following arrangement is used, figure 2. A lens B forms an image of a small source of light P on the galvanometer mirror G . The light passes through a slit A , so placed that it and the template D

are also at conjugate foci of the lens B . Figure 2 shows the general layout of the whole apparatus. Calculations of the optical system, and repeated trials, showed that the following dimensions gave the best performance: $AB = 27$ cm., $BD = 138$ cm., $GD = 66.6$ cm., $DL = 33.3$ cm., $LS = 100$ cm.; the lens L has an aperture 4.5 cm. square, and its two focal lengths are 50 and 100 cm. A further item in the optical system, not previously mentioned, is a strip of cylindrical lens C , which is mounted in the box that houses S , and serves to concentrate the light into a finer spot on the drum. To enable the instrument to be used in its unmodified form if required, the lens L and the template D are mounted together on a geometrical stand, which enables them to be rapidly removed and replaced. The same applies to the double-prism system R , but the items P , A , and B remain permanently in position. When the unmodified form is required, a second source of light, placed near P , is used, and R is replaced by a single totally reflecting prism.

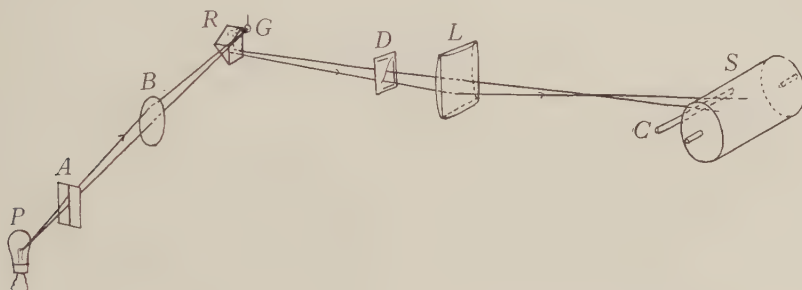


Figure 2. General lay-out of the whole apparatus.

The method of manufacturing the template in the shape of the (intensity, blackening) curve is as follows. A set of density marks is obtained on the plate in the usual way and photometered with the normal form of the instrument, and the (intensity, blackening) curve is drawn on a suitable scale. A piece of aluminium foil is obtained, smoothed out by rubbing on a sheet of glass until the surface has a high polish, and thinly coated with paraffin wax. By means of a pantagraph giving a reduction of 1 : 5 this wax is then scratched off in the shape of the curve, and the foil is immersed in a bath of caustic soda. In about fifteen minutes the foil is etched completely through, leaving an edge which can be surprisingly sharp and smooth. To adjust the instrument for recording intensities, the template is clamped in a holder having screw-controlled traverse and tilting motions, and is moved until the undeflected line of light lies along the line $b = 1$. In order to make this adjustment the more accurately, a part of this line is etched through the foil, as are also parts of the lines $b = 0$ and $I = 0$. The appearance of the template is as shown in the plate at (c), which is a contact print of an actual template. The position of the template having been adjusted, maximum galvanometer deflection is obtained, and the magnitude of this is altered by means of a neutral-tinted wedge in the path of the microphotometer beam until the line of light lies along $b = 0$. The recording then proceeds in the usual way: the spot of light which passes through the slit in the template along $I = 0$ traces out on the record a straight line which gives the

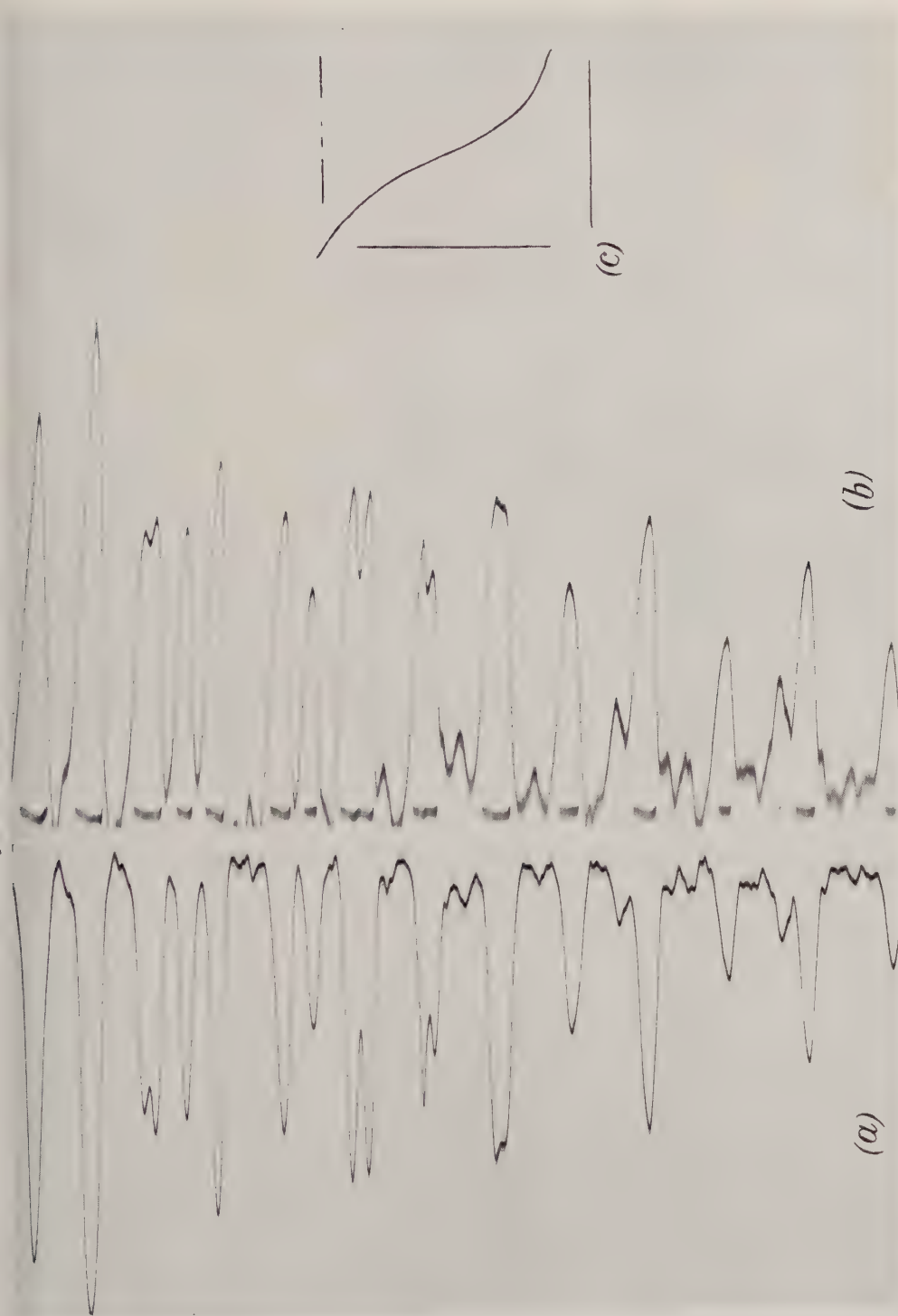
zero from which the intensities are to be measured. It occasionally happens that one template is found to be applicable to a number of photographs taken under similar conditions, but in general a separate template must be prepared for each plate photometered. If it is not desired to preserve the template, results almost as good can be obtained by using smoked glass instead of aluminium foil, with a consequent further saving in time.

The plate shows at (a) and (b) the results obtained by photometering the same negative, first with the normal instrument, and secondly by the above method. In order to test the accuracy of the method, a record of part of the iron-arc spectrum, obtained with the normal photometer, was measured up, and the relative intensities of some forty lines were obtained from a calibration curve. A template was prepared from the same curve and the relative intensities were also obtained directly. The values of the intensities of each line, as obtained by the two methods, were then plotted one against the other. The points should lie on a straight line through the origin, and by choosing suitable scales this line can be made to have a slope of 45° . The points were found to lie very close to the line, over a range of intensities from 1 to 14 in arbitrary units. As a measure of the errors, the root mean square value of the perpendicular distances of the experimental points from the (estimated) line of closest fit was calculated; this had the value 0.073, measured in the same units as the intensities. For purposes of comparison, a third record was taken which was an exact repetition of the first, and the correlation between the intensities as found from these two was found in a similar manner. The root-mean-square value of the deviations in this case, on the same scale as before, was 0.071. These figures, which do not represent unique results, serve to show that the addition of this device to a microphotometer does not in any way lessen the accuracy, which appears, in fact, to be limited mainly by the irregularities due to plate-grain. This same cause renders the adjustment of the maximum deflection a little uncertain, and an error in this direction shows up on the record as a non-coincidence of the two zeros (i) as given by the broken straight line caused by the light passing through the slit along $I = 0$, and (ii) as given by the level of the "troughs" in the record itself: see the plate. If the error is but small, it can be shown that the better approximation is given by using the broken straight line. Further, the accuracy is really greater than the above results would indicate; for it is no longer necessary to be content with peak intensities, and a glance at a record will show that the exact value of an ordinate is much more difficult to determine at a peak than elsewhere.

In conclusion, I would like to express my thanks to Messrs Priest and Ashmore, of Sheffield, who very kindly provided me with the double cylindrical lens used in this work.

DISCUSSION

Mr A. HARVEY. Since the shape of the (intensity, blackening) curve is a function of the wave-length it would follow that the template can only be correct for one particular wave-length. It would be of interest to know the ranges over which the author has succeeded in working with the templates he has used.



Record of a photograph of the perturbations in the (1, 2) band of the negative nitrogen spectrum, obtained (a) with the normal instrument, and (b) with the modified form. The template was inserted in the holder in a reversed position, to make (b) the mirror image of (a). Also, the template (c) used to convert (a) into (b).

Mr J. GUILD. There is no doubt that for many purposes this very ingenious device will prove both useful and efficient. It is desirable, however, to point out that if the spectrum under examination is produced with a prism spectrograph, areas measured on the author's records will not be proportional to energy except over a very short range of spectrum in which the variation of dispersion with wave-length can be neglected to within the order of accuracy aimed at in the work. If a greater spectral range is embraced the ordinates will require to be corrected for dispersion and the curves replotted before their areas are measured. Similar considerations apply to spectra produced with gratings of the echelette or echelon type in which the Fraunhofer "spectra of the first class" (i.e. the diffraction bands produced by a single grating element) are relatively narrow. In fact, in the last-mentioned case, correction for the variation in intensity across the band is necessary however restricted the wave-length range under investigation. I am sure, however, that although it is necessary to bear these limitations in mind there are many cases arising in practice where the author's device will be of great assistance to spectroscopists.

Mr R. S. WHIPPLE. I had the pleasure of seeing the author's device in Sheffield a few days ago, and was much impressed with the simplicity of his apparatus. As the instruments were arranged it was possible to change from one to the other method of recording in the course of a few minutes. I gathered that in practice the author generally uses a template made on smoked glass, and that this is a simple and satisfactory procedure. It greatly simplifies the technique, and may become the method that is generally employed.

AUTHOR'S reply. In reply to Mr Harvey: The range over which a template can be used depends on the nature of the photographic plate and on the region of the spectrum being investigated. The author has made no tests himself, but Lochte-Holtgreven* obtained results which indicate that there would be little variation in the shape of the curve over a range of 60 Å. The range usually quoted in which errors due to this cause can be neglected is 20–30 Å.

While I do not claim, in the case of heterochromatic photometry, to have removed the necessity for correcting for dispersion, the procedure involved is not quite that indicated by Mr Guild. If it can be shown that the experimentally determined (intensity, blackening) curve has the same shape and size over a certain range of wave-lengths, no further correction for the variation of the dispersion in this range need be made. For suppose the dispersion at one end is twice that at the other. With a source of light in which the intensity I_λ is independent of the wave-length λ , the quantity of light per unit area of photographic plate at one end will be half that at the other, and thus the ordinates on the record will be halved. On the other hand, the abscissae will be doubled, on account of the increased dispersion, and thus the area will be unaffected. We might note in passing that in such a case peak intensities would lead to quite erroneous results.

I_λ
 λ

* *Z. f. Phys.* 64, 443 (1930).

If the variation of the experimental (intensity, blackening) curve cannot be neglected, suppose we divide the range into a number of smaller regions in which this approximation can be made. We must now use a standard lamp for which the (I_λ, λ) relation is known. The spectrum of this lamp is photometered across at the mid-points of these sub-regions, and the peak intensities are noted. From the known (I_λ, λ) relation and the known dispersion the amounts of light falling on unit area of the photographic plate at the various points can be calculated. These will not agree with the peak intensities, owing to the varying plate sensitivity, and the ratios of the former to the latter will give a set of factors by which experimentally determined intensities in each sub-region must be multiplied in order to make them comparable with one another.

As was mentioned above, the size of these sub-regions is of the order of 20 Å, large enough to include the whole of a single band of most systems.

A METHOD FOR THE DETERMINATION OF THE THERMAL CONDUCTIVITIES OF ROCKS*

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ABSTRACT. The rock specimens are turned as circular cylinders of diameter 5 cm. and height 2 cm., and are bisected by a cut made perpendicular to the base along one diameter. The top of the cylinder is heated and the temperature-gradient in the specimen is measured by means of thermocouples held in a mica holder inserted in the cut. The temperature-distribution and heat-flow in the specimen are each shown to be represented by a series containing Bessel and hyperbolic functions. Constants involved in the arguments of these functions are shown to be dependent upon the loss of heat from the hot surfaces exposed to the air in the apparatus. The determination of these surface heat-losses is described. Observations and results are given for four specimens.

§ 1. INTRODUCTION

THE work described in this paper was undertaken at the suggestion of Prof. C. H. Lees with the object of developing a method for the determination of the thermal conductivities of rock specimens. It was intended that the apparatus should be of simple design and that the preparation of specimens should not involve drilling. The specimens were turned as circular cylinders about 5 cm. in diameter and 2 cm. in height; they were bisected by a cut along a diameter perpendicular to the plane surfaces.

The top of the specimen was heated, and the temperature-gradient in it in the steady state was measured with two thermocouples placed between the halves of the cylinder. From this gradient, the power input and the value of the total surface emissivity obtained in a separate experiment, the conductivity could be calculated.

§ 2. DESCRIPTION OF APPARATUS

The apparatus is shown in figure 1. It is made of copper, and stands in a wooden box which prevents draughts from affecting the readings. The thermocouples and power leads coming from the apparatus pass from the wooden box through small glass tubes. The base of the cylindrical copper chamber is about 9 cm. in diameter and 0.4 cm. thick. There is a circular, plane-topped, raised portion at the centre of the base upon which the specimen is fitted. The base carries three ebonite plugs *D*, *E*, *F*, each pierced by two small holes through which thermocouple wires are carried out, and the terminals connected to the heating-coil are mounted upon two further ebonite plugs *G*.

* Thesis approved for the Degree of Master of Science in the University of London.

Between the two semicylinders of rock, when they are upon the raised portion of the base, is a mica holder supporting two thermocouples, the wires of which are parallel to the base. A thin sheet of mica is cut to the same length as the perpendicular face of a semicylinder, but slightly higher, with one long edge straight. The best method was to use a safety-razor blade and a steel rule, the mica being laid upon a piece of plane glass. This piece of mica is then laid upon a short length of a metre scale, with the straight edge in contact with a straight glass edge fixed to the wood as shown in figure 2. The wires of the two thermocouples are then placed on the mica, parallel to the straight edge, with their junctions at the centre. The lines on the scale allow this to be done quite accurately by eye. The couples are held in position during the process of construction by small pieces of sealing-wax run on to

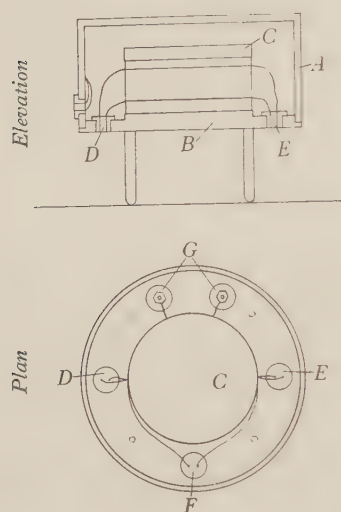


Figure 1.

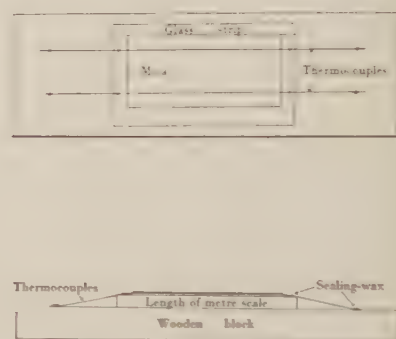


Figure 2.

the wooden holder. Parallelism of the straight edge and the thermocouples having been secured and then checked by means of a travelling microscope, a little seccotine is smeared carefully along each wire to fix it to the mica. This is allowed to harden, the rest of the slip is seccotined, and a second mica slip similar to the first is then laid upon the whole, with its straight edge in contact with the glass edge. A second piece of metre scale is placed upon the mica and a kilogram weight is then placed on the top of this, in order that the holder shall be flat and that excess seccotine shall be forced out. After a day or two such a couple-holder may be removed from the wooden former. The distances of the couples from the straight edge can be readily observed with a travelling microscope. The couple-holder is then gripped firmly in position between the halves of the specimen, with its lower edge in contact with a glass plate on which the halves rest, and its upper edge is trimmed to the height of the specimen by means of the razor blade. The holder is then ready for use.

It is fastened between the halves of the specimen by smearing glycerine upon its surface and the rectangular faces of the specimen. Glycerine is smeared upon the

raised portion of the base and a very thin piece of mica is then placed upon this. The use of this piece of mica is explained below. The base of the specimen is also smeared with glycerine. It is then slid upon the mica. Thermal contact between the specimen and the heating-disc on it is secured in the same way.

The copper heating-disc is a cylindrical box 5 cm. in diameter and 0.5 cm. thick containing a heating coil of nichrome strip wound star-shaped on a mica ring and insulated from the disc with thin mica. The sides of the box are thick so that there is a wide surface of contact between the lid and the sides to ensure good transfer of heat. The lid is held down tightly by six countersunk screws. The ends of the nichrome strip are taken out radially through small glass tubes which just slip over the nichrome. The leads to the coil are of gauge-36 copper wire and are soldered to the nichrome at the points just before they enter the disc. The leads are taken to the two terminals mounted on the base.

The copper cap *A*, which is 0.25 cm. thick and 4 cm. high, fits tightly upon the base *B*. Thermal contact between the cap and the base is made by amalgamating the edges in contact. Thermocouples are soldered to the heating disc and the cap. That on the disc is formed by soldering a copper wire to one end of a diameter and a eureka wire to the other end. The couple fitted to the inside of the cap is placed halfway up and its leads are taken out through a small ebonite plug let into the side of the cap.

Thermocouples. The thermocouples are of gauge-36 copper and eureka wires. The junctions are formed by soldering the scarfed wires together, as fused junctions are very liable to fracture and are, in most cases, larger than scarfed joints. The cold junctions are made in glass tubes containing mercury, immersed in water in a large vacuum-walled 'food jar.' From the cold junctions copper leads run to a four-point mercury switch which enables the various couples to be connected to the potentiometer.

The couples were standardized with their cold junctions in a thermos flask containing ice. The hot junctions were enclosed in very thin glass tubes immersed in succession in the vapours of boiling carbon tetrachloride, water, and aniline and in water at air temperature contained in a thermos flask. The temperature of the water was read with a mercury-in-glass thermometer standardized at the National Physical Laboratory. The e.m.fs. obtained were read on a Cambridge Scientific Instrument Company thermocouple potentiometer and could be read to $1\mu\text{V}$. The e.m.fs. obtained were used to construct curves of deviation from the standard e.m.f. tables given in the *International Critical Tables*. These deviation curves were then used with the standard e.m.f. table to find the temperatures. The couples in one mica slip were calibrated before being placed in the mica, and when in position were compared in an electric furnace with a calibrated thermocouple. At the temperatures used in experiments with the rock specimens the readings were found to be the same as when the couples were enclosed in thin glass tubes for calibration. The couples for the mica holders were therefore calibrated in thin glass tubes before being mounted, as this was more convenient.

Mention has been made of a thin sheet of mica placed between the base of the

apparatus and the base of the specimen. Cross-coupling of the various thermocouples can be effected at the four-point switch, which is used to change from one couple to another and serves to check the insulation of the couple; the latter is generally good when the apparatus is first set up but a leak develops owing apparently to the glycerine hydrolysing on the mercury-copper surface at the base, where traces of a greenish solution appear. As a result, the glycerine on the mica holder served as a poor conductor between the lower couple leads and the base.

On insertion of the thin mica sheet the leak was no longer observed. Mention has been made of this as the author has not seen any previous reference to such an effect.

§ 3. THEORY

v
 z, r, θ We may consider the specimen to be a cylinder as in figure 4, where v represents the temperature-excess of the point z, r, θ over that of the enclosure. Considering a cylinder as above, we have the following boundary conditions.

When	$z = 0$	$v = 0,$
„	$z = b$	$v = V,$
„	$r = a$	$-k \cdot \partial v / \partial r = hv,$

h, k where h, k are respectively the emissivity and the conductivity of the material.

Laplace's equation in cylindrical co-ordinates

$$\frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} + \frac{1}{r^2} \frac{\partial^2 v}{\partial \theta^2} + \frac{\partial^2 v}{\partial z^2} = 0.$$

Since the system is symmetrical about the z axis, the $\partial^2 v / \partial \theta^2$ term is zero.

It can be shown that the solution of this equation is

$$v = A' J_0(\alpha r) \cosh \alpha z + B' J_0(\alpha r) \sinh \alpha z,$$

A', B', α where A', B' and α are constants to be found from the boundary conditions and $J_0(\alpha r)$ is a Bessel function of zero order.

Since $v = 0$ for $z = 0$ the term involving the hyperbolic cosine disappears.

$$\therefore v = B' J_0(\alpha r) \sinh \alpha z.$$

At $r = a,$
$$-\frac{\partial v}{\partial r} = K v,$$

where $K = h/k$, and, since

$$\frac{d}{dx} [J_0(\alpha x)] = -\alpha J_1(\alpha x),$$

we have

$$\alpha a \cdot J_1(\alpha a) / J_0(\alpha a) = Ka,$$

$\alpha_1, \alpha_2, \alpha_3$ which has solutions $\alpha_1, \alpha_2, \alpha_3$ etc. For the specimens used we can show that $\alpha_1 a = 0.33$ approximately and $\alpha_2 a = 3.85$, so that since $a = 2.5$ cm.,

$$\alpha_1 = 0.132, \quad \alpha_2 = 1.53.$$

v will be given by

$$v = \sum_{s=1}^{s=\infty} B_s' \cdot J_0(\alpha_s r) \sinh \alpha_s z,$$

which we may write

$$v = \sum_{s=1}^{s=\infty} B_s \cdot J_0(\alpha_s r) \frac{\sinh \alpha_s z}{\sinh \alpha_s b}.$$

We then have, since $v = V$ when $z = b$,

$$V = \sum B_s J_0(\alpha_s r).$$

It may be shown* that

$$B_s = \frac{2}{a^2} \cdot \frac{1}{K^2 + \alpha_s^2} \cdot \frac{VaK}{J_0(\alpha_s a)}.$$

Hence we have

$$v = \sum_{s=1}^{s=\infty} \frac{2}{a^2} \cdot \frac{1}{K^2 + \alpha_s^2} \cdot \frac{VaK}{J_0(\alpha_s a)} \cdot \frac{J_0(\alpha_s r)}{1} \cdot \frac{\sinh \alpha_s z}{\sinh \alpha_s b}.$$

Dropping suffixes and retaining B ,

$$\frac{\partial v}{\partial z} = \sum \alpha B J_0(\alpha r) \frac{\cosh \alpha z}{\sinh \alpha b}.$$

The heat conducted over the entire cross section at a height z

$$\begin{aligned} &= \int_0^a \sum \alpha B J_0(\alpha r) \frac{\cosh \alpha z}{\sinh \alpha b} \cdot 2\pi r dr \\ &= 2\pi ka \sum B \frac{\cosh \alpha z}{\sinh \alpha b} \cdot J_1(\alpha a). \end{aligned}$$

The heat flow across top surface $z = b$ is

$$2\pi ka \sum B \frac{\cosh \alpha b}{\sinh \alpha b} \cdot J_1(\alpha a).$$

In an experiment we determine v_1 and v_2 at positions z_1 and z_2 , r being zero in both cases, so that

$$v_1 = \sum B \frac{\sinh \alpha z_1}{\sinh \alpha b},$$

$$v_2 = \sum B \frac{\sinh \alpha z_2}{\sinh \alpha b}.$$

The heat flow across the top surface of the specimen is equal to the heat input to the heating-disc less the heat emitted from the top and sides of the disc. If we take V as the temperature-excess of the disc over the cap, S_0 as the area of the top and sides, and h_0 as the value of the total emissivity for the top and sides, the heat flow through the top surfaces is $(H - h_0 S_0 V)$ where H is the heat input to the coil.

V, S_0
 h_0
 H

Hence

$$H - h_0 S_0 V = 2\pi ka \sum B \cdot \frac{\cosh \alpha b}{\sinh \alpha b} \cdot J_1(\alpha a),$$

$$\frac{H - h_0 S_0 V}{v_1 - v_2} = 2\pi ka \frac{\sum B \cdot \frac{\cosh \alpha b}{\sinh \alpha b} \cdot J_1(\alpha a)}{\sum B \cdot \frac{\sinh \alpha z_1 - \sinh \alpha z_2}{\sinh \alpha b}}.$$

* MacRobert, *Spherical Harmonics*, p. 283.

Taking, as a first approximation, the first term in each summation we have

$$\begin{aligned} \frac{H - h_0 S_0 V}{v_1 - v_2} &= \frac{2\pi ka \cdot B_1 \cdot \frac{\cosh \alpha_1 b}{\sinh \alpha_1 b} \cdot J_1(\alpha_1 a)}{B_1 \frac{\sinh \alpha_1 z_1 - \sinh \alpha_1 z_2}{\sinh \alpha_1 b}} \\ &= \frac{2\pi ka \cosh \alpha_1 b \left(\frac{\alpha_1 a}{2} - \frac{\alpha_1^3 a^3}{2 \cdot 8} \right)}{2 \sinh \frac{1}{2} \alpha_1 (z_1 - z_2) \cosh \frac{1}{2} \alpha_1 (z_1 + z_2)} \\ &= \frac{\pi ka \cosh \alpha_1 b \cdot \frac{\alpha_1 a}{2} \cdot \left(1 - \frac{\alpha_1^2 a^2}{8} \right)}{\sinh \frac{1}{2} \alpha_1 (z_1 - z_2) \cosh \frac{1}{2} \alpha_1 (z_1 + z_2)} \dots\dots(1). \end{aligned}$$

When α is small, this reduces to

$$\frac{H - h_0 S_0 V}{v_1 - v_2} = \frac{\pi a^2 k}{z_1 - z_2},$$

giving k thus,

$$k = \frac{H - h_0 S_0 V}{\pi a^2} \cdot \frac{z_1 - z_2}{v_1 - v_2} \dots\dots(2).$$

Having obtained this approximate value of k we can proceed to take into account two terms in each expansion.

$$H - h_0 S_0 V = 2\pi ak \frac{B_1' \cdot \cosh \alpha_1 b \cdot J_1(\alpha_1 a) + B_2' \cdot \cosh \alpha_2 b \cdot J_1(\alpha_2 a)}{B_1' (\sinh \alpha_1 z_1 - \sinh \alpha_1 z_2) + B_2' (\sinh \alpha_2 z_1 - \sinh \alpha_2 z_2)},$$

B_s'

where

$$B_s' = \frac{2}{a^2} \cdot \frac{1}{K^2 + \alpha_s^2} \cdot \frac{Vak}{J_0(\alpha_s a)} \cdot \frac{1}{\sinh \alpha_s b}.$$

Cancelling out the common factor $\frac{2}{a^2} \cdot \frac{Vak}{1}$ we have finally

$$\begin{aligned} \frac{H - h_0 S_0 V}{v_1 - v_2} &= 2\pi ak \frac{\frac{1}{K^2 + \alpha_1^2} \cdot \frac{J_1(\alpha_1 a)}{J_0(\alpha_1 a)} \cdot \frac{\cosh \alpha_1 b}{\sinh \alpha_1 b} + \frac{1}{K^2 + \alpha_2^2} \cdot \frac{J_1(\alpha_2 a)}{J_0(\alpha_2 a)} \cdot \frac{\cosh \alpha_2 b}{\sinh \alpha_2 b}}{\frac{1}{K^2 + \alpha_1^2} \cdot \frac{1}{J_0(\alpha_1 a)} \cdot \frac{\sinh \alpha_1 z_1 - \sinh \alpha_1 z_2}{\sinh \alpha_1 b} + \frac{1}{K^2 + \alpha_2^2} \cdot \frac{1}{J_0(\alpha_2 a)} \cdot \frac{\sinh \alpha_2 z_1 - \sinh \alpha_2 z_2}{\sinh \alpha_2 b}} \\ &\dots\dots(3). \end{aligned}$$

From this equation k may be calculated to a second order of approximation.

Determination of the values of α_s . The quantities $\alpha_1, \alpha_2, \dots$ satisfy the equation

$$\frac{\alpha_s a \cdot J_1(\alpha_s a)}{J_0(\alpha_s a)} = Ka.$$

Using tables of $J_0(x)$ and $J_1(x)$ we can plot values of $x \cdot J_1(x)/J_0(x)$ against x , and then read off the values of x required to give α_1, α_2 , etc. The values necessary for this are given in another paper by the author*. Further,

$$aK = ah/k,$$

* Page 462 of this volume.

and k is readily found from equation (2) while h is found by the method described later.

With the specimens used, the portions of the curve required for the determination of α_1 and α_2 are shown in figures 3*a* and 4*a* of the above-mentioned paper.

§ 4. DETERMINATION OF h

It will be seen that, in the theory given above, the heat-loss in cal./sec. at a surface S is taken as ShV where h is a constant and V is the temperature-excess over that of a surrounding enclosure.

S, h, V

The method of finding h was to support the heating-disc in its usual position in the apparatus by means of a small glass tripod. Thermal contact between the legs of the tripod and the disc, and between the triangle of the tripod and the base of the apparatus, was maintained with glycerine films. Current was then passed through the heating coil and, after about an hour, the readings of the disc and cap thermocouples were taken at 10-minute intervals. When the difference of these readings was constant the final readings were used to find the value of h .

The heat lost from the disc was due partly to conduction down the glass tripod and partly to conduction down the thermocouple wires, in addition to that emitted from the surface by convection and radiation. The first two have, of course, to be allowed for.

In the case of the tripod we have to consider the conduction down three cylinders.

The dimensions of the tripod legs were:

length	$d = 2.2 \text{ cm.},$	d
cross-sectional area	$q = 0.04\pi \text{ cm}^2,$	q
perimeter	$p = 0.4\pi \text{ cm.}$	p

Moreover $h = 0.0003$, $k = 0.004$, while

$$v = Ae^{-(ph/qk) \cdot x} + Be^{-(ph/qk) \cdot x}.$$

At $x = 0$, $v = V$, so that

$$V = A + B \quad \dots\dots(4).$$

At $x = d$, $v = 0$, so that

$$0 = Ae^{-(ph/qk) \cdot d} + Be^{-(ph/qk) \cdot d} \quad \dots\dots(5).$$

$$\text{At } x = 0, \quad \frac{dv}{dx} = \sqrt{\left(\frac{ph}{qk}\right)} (A - B) = 1.23 (A - B) \quad \dots\dots(6).$$

Hence from equations (4) and (5) we can find A and B , and using these values can find dv/dx and hence $-qk \cdot dv/dx$ for each leg of the tripod.

To determine the correction for the heat flow along the thermo-wires we write $v = v_0 e^{-(ph/qk) \cdot x}$, where v is the temperature at a point on the wire distant x cm. from the disc.

v_0, v, x

The heat conducted away from the disc by the wire per second

$$\begin{aligned} &= -qk \cdot dv/dx \quad \text{for } x = 0 \\ &= \sqrt{(qk \cdot ph)} \cdot v_0. \end{aligned}$$

For copper wire of gauge no. 36,

$$\sqrt{qk}.ph = 1.1 \times 10^{-4}.$$

For eureka wire of gauge no. 36,

$$\sqrt{qk}.ph = 0.3 \times 10^{-4}.$$

The total loss along the thermocouple leads is $1.4 \times 10^{-4} v_0$ cal./sec.

The heat input having been corrected by these means the value of h is given by

$$h = \frac{\text{Corrected heat input}}{(\text{Disc temperature} - \text{Cap temperature}) \cdot \text{Total surface area of disc}}.$$

Readings were taken for various heat inputs. The results are shown graphically in figure 3. It will be seen that h is not constant but increases with increase of temperature-difference. Consequently in the calculation of k the value of h is read from the curve for a temperature-excess equal to the mean of the temperature-excesses indicated by the thermocouples in the mica slip. The error introduced by this procedure will be small, since equation (2) which gives k fairly accurately is independent of α .

§5. HEAT-LOSS FROM TOP AND SIDES OF HEATING-DISC

The value of h obtained by means of the above experiment is a mean of the values of the total emissivities of the top, bottom and sides of the disc. It probably corresponds very closely to the value for the sides, as it has been shown* that the ratios of the heat-losses by convection from the top, sides and bottom of a body in free air are roughly as $1 : \frac{3}{4} : \frac{1}{2}$. The value of h_0 , i.e. the mean total emissivity of the top and sides, will of course be greater than the value of h which is found by the tripod method. Some idea of it may be obtained from the following considerations.

If we have a surface S with a temperature-excess V over that of an enclosure it will emit heat from the surface by radiation and convection. Then the heat emitted

$$= S [\sigma (T_1^4 - T_0^4) + hV],$$

T_1 and T_0 being the absolute temperatures of the surface and the enclosure, respectively; and

$$T_1 - T_0 = V.$$

Taking the figure for the emissivity of a dull copper surface, given by Griffiths and Davis*, we have

$$\sigma = 0.15 \times 1.36 \times 10^{-12}.$$

In an experiment with the disc resting on a glass tripod we have heat being emitted from the top, sides and bottom, and, as has been said above, it has been shown that the values of the respective convection losses will be roughly as $1 : \frac{3}{4} : \frac{1}{2}$. Taking them as h , $\frac{3}{4}h$, $\frac{1}{2}h$ we have, in a tripod experiment, if S_1 is the horizontal surface and S_2 is the vertical surface, that the heat emitted

$$= S_1 [\sigma (T_1^4 - T_0^4) + hV] + S_2 [\sigma (T_1^4 - T_0^4) + \frac{3}{4}hV] + S_1 [\sigma (T_1^4 - T_0^4) + \frac{1}{2}hV]$$

$$= \sigma (T_1^4 - T_0^4) (2S_1 + S_2) + \frac{3}{4}h (2S_1 + S_2) V.$$

This in our calculations we put equal to $(2S_1 + S_2) h''V$, obtaining

$$h'' = [\sigma (T_1^4 - T_0^4) + \frac{3}{4}hV]/V.$$

* Food Investigation Board, Special Report, no. 9.

From the top and sides of the disc would be emitted at the same time

$$S_1 [\sigma (T_1^4 - T_0^4) + \bar{h}V] + S_2 [\sigma (T_1^4 - T_0^4) + \frac{3}{4}\bar{h}V].$$

To find h_0 , the mean value of the total emissivity for the top and sides, we put this equal to $(S_1 + S_2) h_0 V$ so that

$$h_0 = \{S_1 [\sigma (T_1^4 - T_0^4) + \bar{h}V] + S_2 [\sigma (T_1^4 - T_0^4) + \frac{3}{4}\bar{h}V]\} / (S_1 + S_2) V \\ = [\sigma (T_1^4 - T_0^4) (S_1 + S_2) + \bar{h}V (S_1 + \frac{3}{4}S_2)] / (S_1 + S_2) V.$$

$(T_1^4 - T_0^4)$ can be put equal to $4T_0^3(T_1 - T_0)$ if $(T_1 - T_0)$ is small compared with T_0 .

In the experiments performed T_0 is approximately 290°K , so that

$$\sigma (T_1^4 - T_0^4) = 2 \times 10^{-5} \times V$$

approximately. Hence

$$h_0 = 2 \times 10^{-5} + \bar{h} (S_1 + \frac{3}{4}S_2) / (S_1 + S_2), \\ h'' = 2 \times 10^{-5} + 0.75\bar{h}.$$

Substituting in the expression for h_0 the values of S_1 and S_2 for the disc used we have

$$h_0 = 2 \times 10^{-5} + 0.9\bar{h}.$$

Taking a value of h'' from the curve obtained from experiments with the disc upon the glass tripod we have at

$$V = 6^\circ \text{C.}, \quad h'' = 1.3 \times 10^{-4}, \\ h'' = 2 \times 10^{-5} + 0.75\bar{h} = 1.3 \times 10^{-4}, \\ \therefore \bar{h} = 1.5 \times 10^{-4}, \\ \therefore h_0 = 2 \times 10^{-5} + 0.9\bar{h} = 1.6 \times 10^{-4},$$

i.e. h_0 is approximately 25 per cent larger than h'' .

One might, therefore, obtain values of h_0 by adding 25 per cent to the values of h'' obtained from the tripod experiments. Any inaccuracies due to approximations in the above calculations would be of small importance, for with the specimens used the heat emitted from the heating-disc is only about 5 per cent of the heat input to the specimen. An error of 10 per cent in the value of h_0 would, therefore, result in an error of about $\frac{1}{2}$ per cent in the value of k . It was thought that some experimental test of this view should be carried out. The method adopted was to coat the lower horizontal surface of the disc and the raised portion of the base of the apparatus with lampblack. This was done by holding them over the flame of turpentine burning on a piece of asbestos wool. A cylindrical paper tube was then constructed from drawing-paper. This tube was of the same height as the rock specimen and had a diameter slightly smaller than that of the disc. One edge of the tube was seccotined lightly and it was fixed to the base of the disc. The tube, with heating-disc fixed on the top of it, was then placed in position, as if it were a specimen, upon the raised portion of the base of the apparatus. The cap was, of course, fitted, current was passed through the heating-coil, and readings of the disc and cap couples were taken. It was found that readings could be taken fairly rapidly by raising the temperature quickly to a desired point and then reducing the current to the value calculated to give that temperature, readings then proceeding at 10-minute intervals for about 40 minutes.

The heat emitted from the top and sides was found by subtracting from the heat input to the coil, the heat conducted away by the cylinder of air, and that radiated from the base of the disc. The correction for the heat conducted down the cylinder of air was obtained by means of a calculation similar to those for the glass tripod, it being assumed that convection was absent when the heating-disc was uppermost. The heat radiated to the base was calculated from the formula

$$S\sigma(T_1^4 - T_0^4),$$

σ being taken for the lampblack surface as 0.9 of the value for a black body radiator.

The values of h_0 thus obtained are shown on the curve in figure 3. It will be seen that the values obtained lie on a curve which approximates to "a 25 per cent correction to h'' " curve.

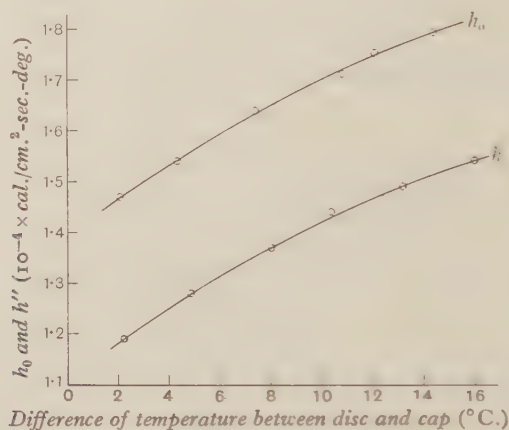


Figure 3. Values of h_0 and h'' .

As the approximate calculations support the results obtained from the above experiments, the value of h_0 used in the calculation of the heat input to the specimen in the steady-state experiments was read off the h_0 curve obtained from those experiments.

§ 6. FULFILMENT OF THE CONDITIONS ASSUMED IN THE THEORY GIVING THE TEMPERATURE-DISTRIBUTION IN THE STEADY STATE

The theory given postulates a cylinder with an isothermal upper surface. The heating-disc, being of copper of fairly thick section and having quite a small diameter, complies with this condition. The other boundary condition, that the temperature of the base is equal to that of the enclosure, was tested by fitting another thermocouple to the base of the apparatus. As the apparatus was first constructed the contact between the cap and base was copper-to-copper; with this arrangement differences of temperature were observed. Upon amalgamating the surfaces of contact of cap and base these differences were greatly reduced and could be ignored at the temperatures used.

There is also the question of the effect of the couple-holder. The couple-holder, being surrounded by a much larger volume of rock, will tend to attain closely the temperature-distribution of the larger volume. Any slight differences would affect the reading of each couple in the holder by the same amount and the resultant temperature-gradient would remain unaltered. That the effect is extremely small was confirmed by the readings taken in some earlier experiments. In these the two semicylinders were cemented together with Keene's cement, thermocouples being embedded in the cement. These cement couple-holders were quite thick, up to 2 mm., and of course the thermal conductivity of the cement is different from that of mica. However, the temperature-gradients obtained with these cement holders were, to a close degree of accuracy, the same as those obtained with the mica couple-holders. Also, for a given specimen, the same values of k are obtained from observations taken with different mica couple-holders.

The influence of the mica couple-holder upon the quantity of heat flowing into the specimen, and the effect of the specimen not having a circular section, must also be considered. The heating disc is circular but the specimens are, of course, not

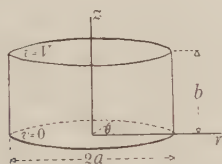


Figure 4.

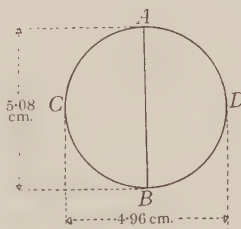


Figure 5.

truly cylindrical. They were originally turned as circular cylinders but when they were bisected the portion of the rock lost in the cutting rendered the specimen no longer cylindrical. Consequently the heating-disc has a small part of its lower surface which is not in contact with the specimen. Heat is emitted from this portion of the base of the disc to the enclosure.

In addition, there is the fact that the arrangement of specimen and slip constitutes two thermal resistances in parallel. In the case of specimen no. 3, figure 5, the heating-disc has a diameter of 5.08 cm., the couple-holder has a thickness of 0.04 cm., the area of the top of the slip is 0.20 cm², the area of the heating-disc is 20.27 cm², and the area of the specimen is 19.66 cm². The area of the lower surface of the heating-disc not in contact with the specimen is 0.41 cm². Taking a temperature-excess of 8° C. and $h = 0.00013$ we have that heat is emitted from this area at the rate of 4×10^{-4} cal./sec. At this temperature-excess the heat input to the specimen is about 0.5 cal./sec. The correction which would have to be applied is small and is, therefore, neglected in practice. If it be assumed that $k = 5 \times 10^{-3}$ for the specimen and $k = 1 \times 10^{-3}$ for the couple-holder, the thermal resistance of 1 cm. of the specimen is 1/0.098 and of the couple-holder 1/0.0002.

The heat flow down the couple-holder would therefore be about $\frac{1}{5}$ per cent. In

a the calculations to find k it is neglected. In the finding of k from a set of steady-state observations the mean of the lengths AB and CD is taken as the value of a , the radius of the cylinder.

§7. EXPERIMENT AND CALCULATIONS TO FIND k

The apparatus is set up as described. The current is supplied by secondary cells. The current passing through the heating-coil is measured by finding the potential-drop across a resistance of 0.1Ω in series with the heating-coil. This potential-drop is measured on the thermocouple potentiometer. The potential-difference at the terminals on the case is obtained by means of a potential-divider which reduces the p.d. to one thousandth of the value at the terminals. The resulting p.d. is then read on the thermocouple potentiometer. The power input at the coil terminals can thus be found accurately. The value of the heat input to the disc is obtained by correcting for the power lost in the copper leads and the heat flow along the leads of the disc thermocouple.

A high resistance, which can be smoothly changed by small amounts, is connected in parallel with the resistance regulating the current. This high resistance acts as a convenient vernier control for keeping a constant current. Small current-variations can be easily observed on the potentiometer and corrected with this device.

When the current has passed for about 2 hours, readings of the four thermocouples are commenced at 10-minute intervals. These are continued until the steady state is reached; this takes from about 1 to 3 hours, depending upon the power input. When the steady state has been reached the final readings then taken are used to calculate k . One such set of calculations is given for one specimen.

Specimen no. 3. Gabbro.

Current (A.)	e.m.f. (V.)	Thermocouple readings (mV.)				Cold-junction temperature (°C.)
		Disc	Upper	Lower	Cap	
0.807	4.96	1.733	1.593	1.205	1.066	16.82
Thermocouple temperatures (°C.)						
		56.21	53.15	44.53	41.41	
Positions of thermocouples (distance from base, cm.)						
		1.911	1.524	0.322	0.00	
Heat input.						

Current, 0.807 A., e.m.f. at case terminals, 4.96 V.

Heat input to coil, 0.947 cal./sec.

Heat-loss along thermocouples, 0.002 cal./sec. when $T = 14.80^\circ \text{C}$.

Heat emitted from top and sides, 0.074 cal./sec. when $T = 14.80^\circ \text{C}$. and $h_0 = 0.00018$.

Heat input to specimen, 0.871 cal./sec.

Approximate value of k , 6.2×10^{-3} .

Calculation of α_1 and α_2 .

When $V = 7.43^\circ \text{C.}, \quad h'' = 1.35 \times 10^{-4},$
 $\therefore K = 2.18 \times 10^{-2},$
 $\therefore aK = 5.47 \times 10^{-2}.$

From the curves given in figures 3a and 4a we have,

$\alpha_1 a = 0.330, \quad \text{so that} \quad \alpha_1 = 0.132;$
 $\alpha_2 a = 3.85, \quad \text{so that} \quad \alpha_2 = 1.53.$

Equation (3) may be simplified by certain approximations.

$$\frac{H - h_0 S_0 V}{v_1 - v_2}$$
$$= 2\pi ka \frac{\frac{1}{K^2 + \alpha_1^2} \cdot \frac{J_1(\alpha_1 a)}{J_0(\alpha_1 a)} \cdot \frac{\cosh \alpha_1 b}{\sinh \alpha_1 b} + \frac{1}{K^2 + \alpha_2^2} \cdot \frac{J_1(\alpha_2 a)}{J_0(\alpha_2 a)} \cdot \frac{\cosh \alpha_2 b}{\sinh \alpha_2 b}}{\frac{1}{K^2 + \alpha_1^2} \cdot \frac{1}{J_0(\alpha_1 a)} \cdot \frac{\sinh \alpha_1 z_1 - \sinh \alpha_1 z_2}{\sinh \alpha_1 b} + \frac{1}{K^2 + \alpha_2^2} \cdot \frac{1}{J_0(\alpha_2 a)} \cdot \frac{\sinh \alpha_2 z_1 - \sinh \alpha_2 z_2}{\sinh \alpha_2 b}}$$
$$= 2\pi ka \frac{J_1(\alpha_1 a) \cdot \cosh \alpha_1 b + \frac{K^2 + \alpha_1^2}{K^2 + \alpha_2^2} \cdot \frac{J_0(\alpha_1 a)}{J_0(\alpha_2 a)} \cdot \frac{\sinh \alpha_1 b}{\sinh \alpha_2 b} \cdot \frac{J_1(\alpha_2 a)}{1} \cdot \frac{\cosh \alpha_2 b}{1}}{\sinh \alpha_1 z_1 - \sinh \alpha_1 z_2 + \frac{K^2 + \alpha_1^2}{K^2 + \alpha_2^2} \cdot \frac{J_0(\alpha_1 a)}{J_0(\alpha_2 a)} \cdot \frac{\sinh \alpha_1 b}{\sinh \alpha_2 b} \cdot \frac{(\sinh \alpha_1 z_1 - \sinh \alpha_2 z_2)}{1}}$$

The second terms in both numerator and denominator are small compared with the first term, that in the numerator being about $\frac{1}{40}$ per cent and that in the denominator 2 per cent of their respective first terms. We can, therefore, write

$$J_0(\alpha_1 a)/J_0(\alpha_2 a) = -2.4,$$

since $J_0(\alpha_1 a)$ is always nearly 0.97 and $J_0(\alpha_2 a) = -0.4$. The second term in the numerator is negligible.

That in the denominator may be written

$$\frac{\alpha_1^2}{\alpha_2^2} (-2.4) \frac{\sinh \alpha_1 b}{\sinh \alpha_2 b} (\sinh \alpha_2 z_1 - \sinh \alpha_2 z_2).$$

We have

$$J_1(\alpha_1 a) = \frac{1}{2} \alpha_1 a \left\{ 1 - \frac{1}{2} \left(\frac{\alpha_1 a}{2} \right)^2 \right\};$$

$$\therefore \frac{H - h_0 S_0 V}{v_1 - v_2}$$
$$= \pi a^2 k \alpha_1 \frac{\left\{ 1 - \frac{1}{2} \left(\frac{\alpha_1 a}{2} \right)^2 \right\} \cosh \alpha_1 b}{\left\{ \sinh \alpha_1 z_1 - \sinh \alpha_1 z_2 - 2.4 \left(\frac{\alpha_1}{\alpha_2} \right)^2 \cdot \frac{\sinh \alpha_1 b}{\sinh \alpha_2 b} \cdot (\sinh \alpha_2 z_1 - \sinh \alpha_2 z_2) \right\}}.$$

This is the equation used for the determination of k .

Proceeding with the calculation for specimen no. 3, we have that

$$\frac{H - h_0 S_0 V}{v_1 - v_2} = \frac{\pi \cdot 2.51^2 \cdot k \cdot 0.132 \{1 - 0.014\} 1.032}{\{0.202 - 0.042 - 0.002\}},$$

whence $k = 6.0 \times 10^{-3} \text{ cal./cm.-sec.-deg.}$

§ 8. REVIEW OF RESULTS

The accuracy of the method is chiefly dependent upon the measurement of the distance apart of the couples in the mica holder. In practice the setting up was not considered accurate enough if, when a travelling microscope was moved along the length of the wire parallel to the long edge of the holder, the point of intersection of the cross wires moved off the width of the thermocouple wire. The holders used fulfilled this condition, and as the diameter of the wire was 0.019 cm. the error in distance apart of the couples can be considered to have a maximum of about 0.01 cm. As the distance between the couples was usually about 1.1 to 1.2 cm. one can say that an error of less than 1 per cent is involved on account of the setting up. The final results can, therefore, be considered to be accurate to within 1 per cent.

The method has, of course, the advantage that the error due to the difference of temperature set up in the glycerine layer maintaining thermal contact between the hot and cold plates and the specimen is eliminated.

The rocks used were gabbro, granite, quartz-schist and recrystallized sandstone. The sources were as follows:

Gabbro: from Sligachan, Skye.

Granite: taken at a depth of 270 ft. from the surface at the Newmay Quarry, Aberdeenshire.

Quartz-schist: from Moine, Sutherland.

Recrystallized sandstone: Lower Permian, The Old Quarry, Penrith, Cumberland.

In the case of sandstone a different treatment had to be adopted in the fixing of the mica couple-holder in the specimen, as the sandstone was permeable to the glycerine. Fairly thick shellac varnish was used in place of the glycerine.

The values of the thermal conductivities found with the apparatus are given in the table.

Specimen	Current (A.)	E.M.F. (V.)	Thermocouple readings (mV.)				Cold- junction tempe- rature (°C.)	Den- sity (gm./ cm ³)	Distances from base (cm.)				Mean temp- erature of speci- men (°C)
			Disc	Upper	Lower	Cap			Disc	Upper	Lower	Cap	
Gabbro	0.600	3.71	1.049	0.951	0.765	0.688	16.18	3.10	1.91	1.41	0.34	0.00	36.2
	0.818	5.00	1.793	1.615	1.263	1.115	17.10						49.9
Granite	0.607	3.69	0.922	0.860	0.700	0.635	15.54	2.58	1.91	1.52	0.32	0.00	33.7
	0.813	4.99	1.633	1.515	1.217	1.093	15.72						47.0
Quartz-schist	0.605	3.73	0.883	0.828	0.703	0.650	16.42	2.58	1.91	1.41	0.34	0.00	34.2
	0.798	4.98	1.410	1.312	1.092	0.996	15.96						43.6
Sandstone, recrystallized	0.608	3.70	0.978	0.912	0.802	0.723	16.30	2.40	1.91	1.49	0.41	0.00	36.5
	0.793	4.86	1.509	1.382	1.200	1.056	16.86						46.6

Comparison with the results obtained by other observers is rather difficult. Widely differing values have been observed for specimens of the same name but which may vary considerably in their composition.

Gabbro was used by Tadakora*. He found the conductivities of hornblende gabbro taken from two provinces in Japan. For a specimen from the Province of Chikuzen the value was 0.0072 while from the Province of Awadi the value was 0.0043, both taken at room temperatures.

Granite was used by Poole† and he gives a value of 6×10^{-3} . The present values are much higher, but this is probably due to difference in composition. The amount of quartz, which probably governs the conductivity by both the quantity present and its orientation in the specimen, can, in granite, vary by from 20 per cent to 50 per cent. Poole does not state the source of his specimen.

Quartz-schist is a "sheared quartzite." In this, the quartzite has been crushed by folding movements and the quartz consists of small crystalline fragments of irregular shape with interlocking margins. Some white mica is also contained in crystalline flakes. One might, therefore, expect lower values than those obtained for quartzite. Ensor‡ gives the thermal conductivity of quartzite at 0° C. as 0.0149, which agrees with this expectation.

Previous values for sandstone are considerably lower than the values found, being about 3×10^{-3} at air temperatures. The specimen used in this work is of recrystallized sandstone. The *Encyclopaedia Britannica* states in the section on quartzites that "where sandstones are altered by intrusive rocks they are often converted into pure quartzite, the heat evidently occasioning the deposit of interstitial quartz." Recrystallized sandstone undergoes such forces in its formation, and it is interesting to note that the values of the thermal conductivity observed are of the same order as those for quartzite.

The directions of the variations in the conductivities at the different temperatures agree with those found by previous observers. On account of the limited range no value of a temperature coefficient of thermal conductivity can be given.

§ 9. ACKNOWLEDGMENT

In conclusion, the author would like to express his thanks to Prof. C. H. Lees, F.R.S., for his suggestions and encouragement throughout the course of this work.

* *Science Reports of Tôhoku University*, **10**, 339 (1921).

† *Phil. Mag.* **27**, 82 (1914).

‡ *Proc. Phys. Soc.* **43**, 590 (1931).

TABLES TO FACILITATE THE CALCULATION OF THE TEMPERATURE-DISTRIBUTION IN A CYLINDER

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ABSTRACT. The temperature-distribution, in the case of a cylinder heated at its top surface and whose convex surface is exposed to air at the same temperature as that of the base of the cylinder, has been considered by Byerley. The present paper gives curves and a table which can be used for finding constants occurring in the expression for the temperature-distribution.

THE cases in which the temperature-distribution in a circular cylinder symmetrically heated may be expressed in terms of Bessel functions may be classified, in accordance with Byerley's* suggestions, as follows.

a, b (a) The convex surface and base of a cylinder of radius a and length b are kept at zero temperature, and the temperature at each point of the top surface is a given
 r function of the distance r of the point from the centre of the surface. (b) The base is kept at zero temperature and the convex surface is impervious to heat, the top surface being as in (a). (c) The base is kept at zero temperature and the convex surface of the cylinder is exposed to a gas at zero temperature, the top surface being as in (a).

The solution of each of these problems is given by

$$\mu = A_1 \frac{\sinh(\mu_1 z)}{\sinh(\mu_1 b)} \cdot J_0(\mu_1 r) + A_2 \frac{\sinh(\mu_2 z)}{\sinh(\mu_2 b)} \cdot J_0(\mu_2 r) + \dots$$

z Where J_0 is the Bessel function of zero order, z the height of a point above the
 A_1, A_2, \dots base, A_1, A_2, \dots constants to be determined by the distribution of temperature
 μ_1, μ_2, \dots over the upper surface, and μ_1, μ_2, \dots are in each case the roots of an equation, namely

for (a) $J_0(\mu a) = 0,$

for (b) $J_1(\mu a) = 0,$

for (c) $k \mu a J_1(\mu a) - ah J_0(\mu a) = 0,$

h, k where h is the emissivity of the surface and k is the thermal conductivity of the material of the cylinder.

* W. E. Byerley, *Fourier Series and Spherical Harmonics*, p. 226 (1902).

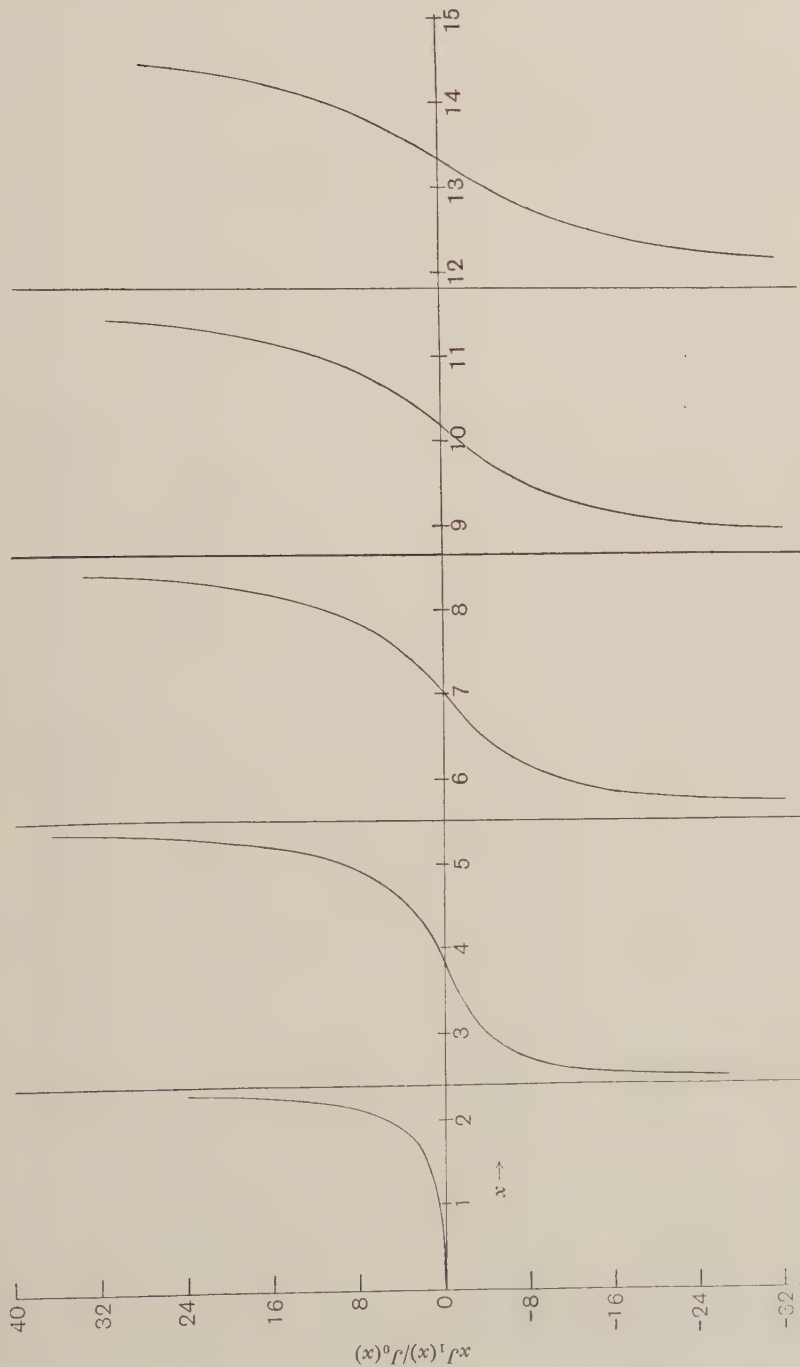


Figure 1.

The roots of (a) and (b) have been tabulated*, and the lower values are reproduced in table 1.

Table 1.

Roots of $J_0(x) = 0$	2.405, 5.520, 8.654, 11.792, 14.931, 18.071, 21.212
Roots of $J_1(x) = 0$	3.832, 7.016, 10.173, 13.324, 16.471, 19.616, 22.760

There appear to be no tables of the roots of equation (c). As this case frequently occurs in practice, the author is following the suggestion of Prof. C. H. Lees, that tables and curves for the determination of its roots would be useful.

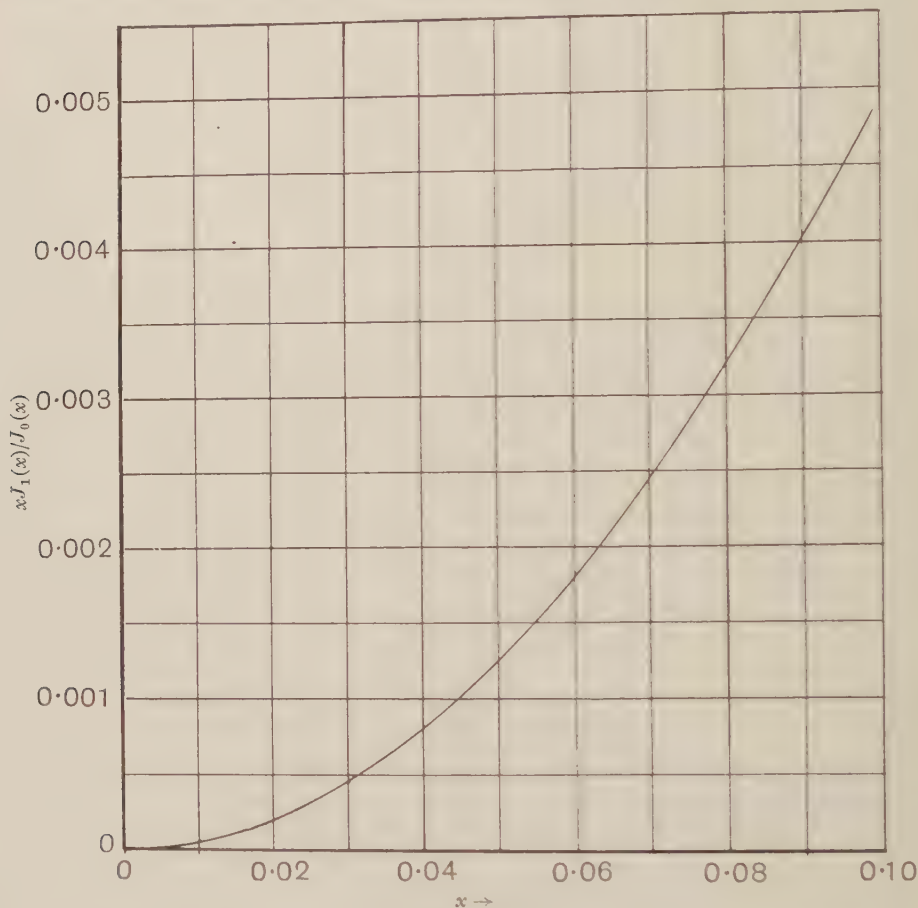


Figure 2.

The equation may be written

$$x, K \quad \frac{x J_1(x)}{J_0(x)} = K, \quad \text{where } x = \mu a \quad \text{and} \quad K = ah/k.$$

If, therefore, values of $x J_1(x) / J_0(x)$ are calculated from tables of $J_1(x)$ and

* G. N. Watson, *Theory of Bessel Functions*, p. 666 (1922); Jahnke, *Funktionentafeln*, p. 111 (1909).

$J_0(x)$ and plotted against x ; the values of x which make $xJ_1(x)/J_0(x)$ equal to K may be found by interpolation or by inspection of the curves. The values of $xJ_1(x)/J_0(x)$ are given in table 2 over the range $x = 0$ to $x = 15$ at small intervals within the ranges likely to be required in practice.

The values when plotted give the curve shown in figure 1. It is, of course, discontinuous, with breaks occurring at the roots of $J_0(x) = 0$, and crosses the x axis at the roots of $J_1(x) = 0$. Each portion gives a value of μ satisfying the equation. The portions over which small intervals have been taken are shown on an enlarged scale in figures 2, 3, 4, 5, 6 and 7. Figures 3*a* and 4*a* are portions of the curve used in work* with some rock specimens.

Table 2.

x	$\frac{xJ_1(x)}{J_0(x)}$	x	$\frac{xJ_1(x)}{J_0(x)}$	x	$\frac{xJ_1(x)}{J_0(x)}$
0.00	0.00	2.20	11.083	8.00	10.936
.01	.00005	2.4048	∞	8.20	17.310
.02	.00020	2.50	- 25.64	8.40	32.890
.03	.00045	2.75	- 7.137	8.6537	∞
.04	.00080	3.00	- 3.911	9.00	- 24.442
.05	.00125	3.25	- 2.355	9.25	- 13.125
.06	.00180	3.50	- 1.268	9.50	- 7.795
.07	.00245	3.75	- 0.310	9.75	- 4.393
.08	.00320	3.8317	0.000	10.00	- 1.769
.09	.00405	3.85	0.0703	10.1735	0.000
.10	.00501	3.90	0.2644	10.20	0.2705
.15	.0113	3.95	0.4624	10.25	0.7830
.20	.0201	4.00	0.6651	10.30	1.3053
.25	.0315	4.05	0.8738	10.35	1.8305
.30	.0455	4.10	1.0894	10.40	2.3704
.35	.0622	4.15	1.3132	10.60	4.7138
.40	.0816	4.20	1.5464	10.80	7.5563
.45	.1039	4.40	2.6069	11.00	11.360
.50	.1291	4.60	3.9850	11.20	17.167
.55	.1573	4.80	5.9592	11.40	28.111
.60	.1886	5.00	9.2222	11.60	60.314
.65	.2243	5.10	11.967	11.7915	∞
.70	.2614	5.20	16.182	12.00	- 56.21
.75	.3031	5.40	45.252	12.25	- 24.32
.80	.3487	5.5201	∞	12.50	- 14.08
.85	.3984	5.75	- 24.06	12.75	- 8.448
.90	.4524	5.85	- 16.58	13.00	- 4.418
.95	.5112	6.00	- 11.03	13.25	- 0.981
1.00	.5751	6.25	- 6.474	13.3237	0.000
1.05	.6444	6.50	- 3.843	13.35	0.3511
1.10	.7197	6.75	- 1.873	13.40	1.0216
1.15	.8018	7.00	- 0.109	13.45	1.7000
1.20	.8909	7.0156	0.000	13.50	2.3892
1.25	.9882	7.05	0.2421	13.55	3.0937
1.30	1.0944	7.10	0.5971	13.60	3.8160
1.35	1.2117	7.15	0.9577	13.80	6.9886
1.40	1.3385	7.20	1.3257	14.00	10.916
1.45	1.4792	7.25	1.7028	14.20	16.333
1.50	1.6351	7.30	2.0913	14.40	25.022
1.60	1.9976	7.35	2.4935	14.60	42.998
1.80	3.0789	7.40	2.9116	14.80	112.92
2.00	5.2719	7.60	4.8092	14.9309	∞
2.10	7.1627	7.80	7.2910		

* See page 447 of this volume.

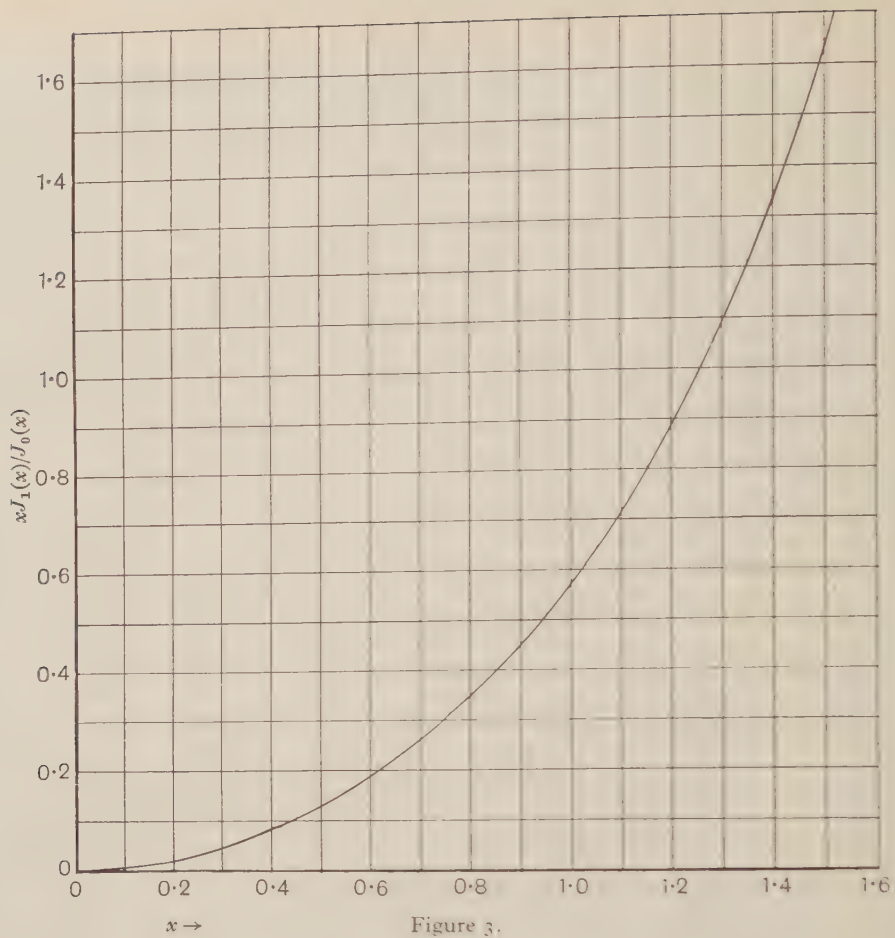


Figure 3.

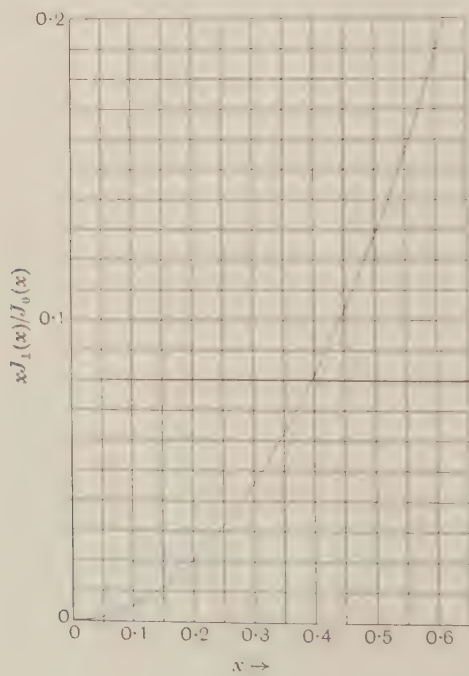


Figure 3 a.

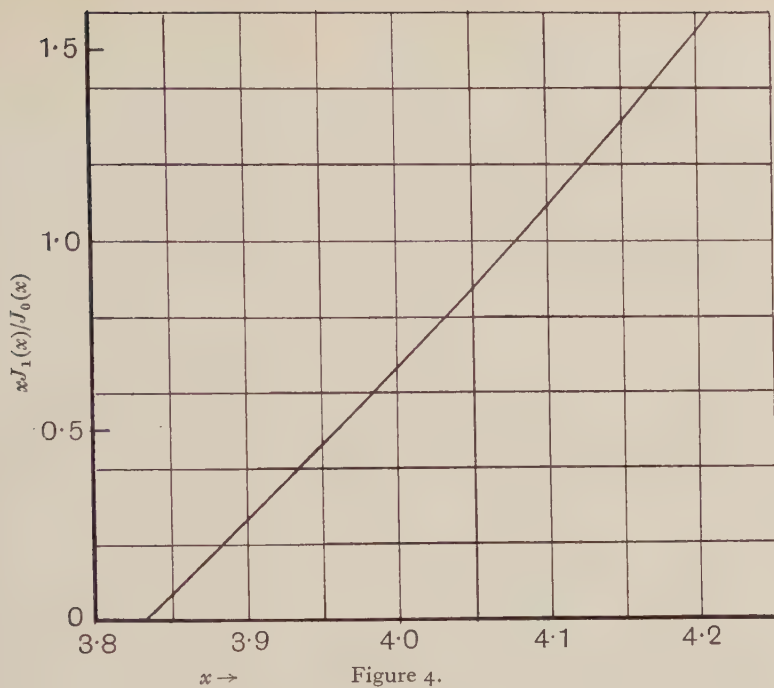


Figure 4.

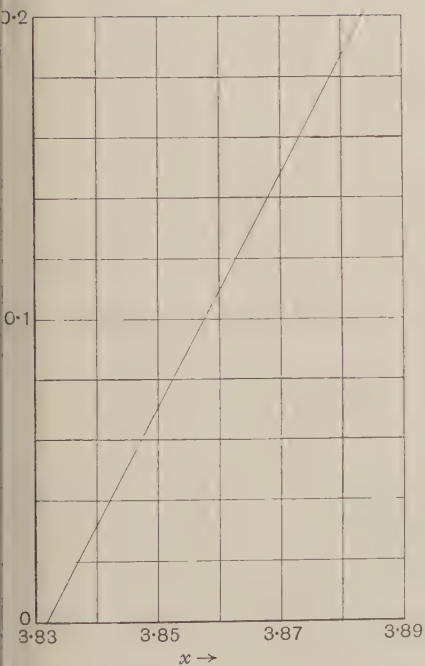


Figure 4a.

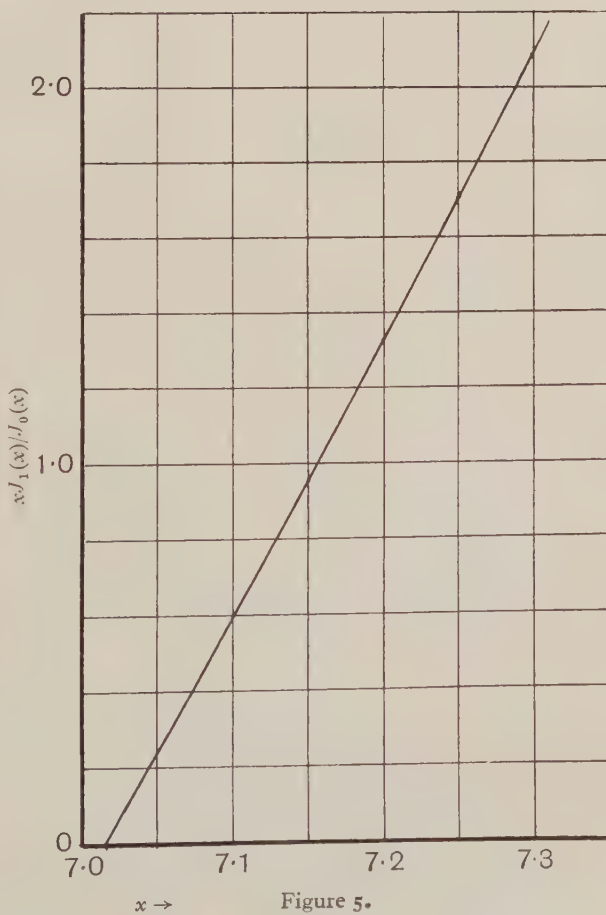


Figure 5.

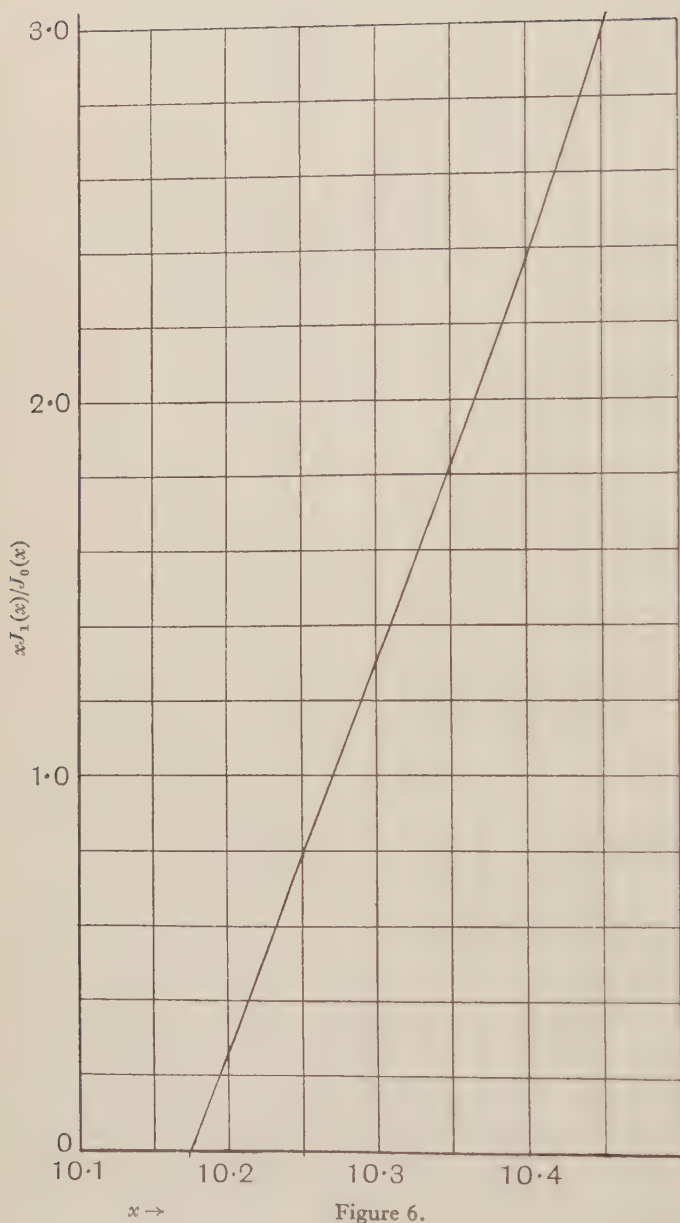


Figure 6.

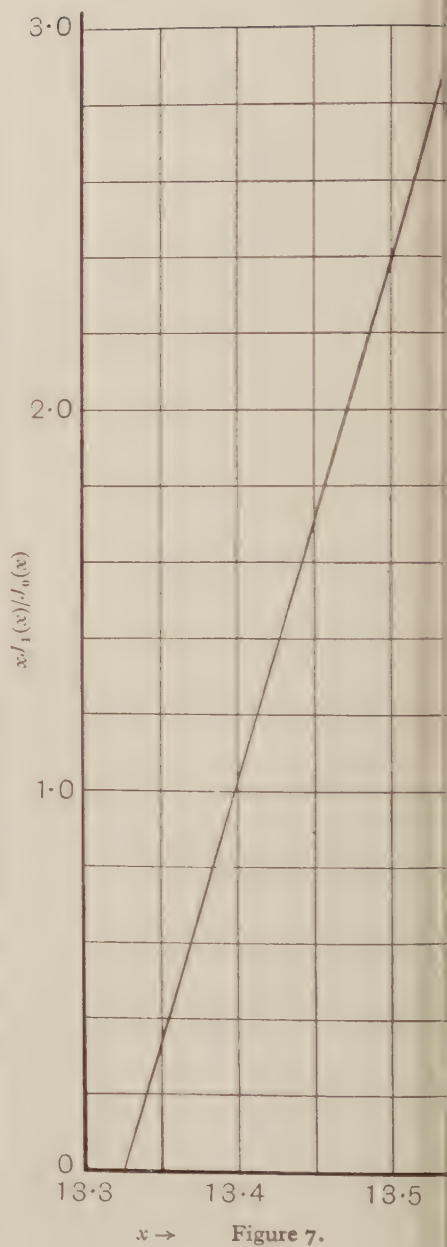


Figure 7.

A NOTE ON THE KERR CELL

BY E. E. WRIGHT, B.Sc., Baird Television Limited.

Communicated by Prof. G. Temple, December 31, 1932. Read March 17, 1933.

ABSTRACT. The distortion due to the curvature of the (light, voltage) characteristic of a Kerr cell is discussed, and an expression giving the amplitudes of the Fourier components of the light-variation due to a pure alternating potential applied to the cell is obtained. Sets of curves showing the variation of percentage of second and third harmonic with bias and amplitude of the applied alternating potential are given. The working conditions for minimum distortion obtained from these curves agree with those used in practice.

§ 1. INTRODUCTION

THE equation of the (light, voltage) characteristic of a Kerr cell* is

$$L = \sin^2 \left\{ \frac{\pi}{2} \left(\frac{e}{V} \right)^2 \right\} \quad \dots\dots(1),$$

where L is the intensity of the light passed by the cell, e the potential applied to the cell, and V the potential at which L is first a maximum. V is a constant for the cell.

L, e
 V

It is obvious from this equation that the light-variations are not proportional to the variations of the applied voltage, although by working on a suitable part of the characteristic and keeping the range of variation of e small the changes in L may be made nearly proportional to the changes in e .

It is the object of this note to investigate this distortion due to the curvature of the characteristic.

§ 2. THEORY

If a pure alternating potential $a \cos \omega t$ superimposed on a steady biasing potential b is applied to the cell, the light-intensity is given by

a, ω, t
 b

$$L = \sin^2 \left\{ \frac{\pi}{2} \left(\frac{b + a \cos \omega t}{V} \right)^2 \right\} \quad \dots\dots(2).$$

Now L is clearly an even function of time, hence

$$L = A_0 + \sum_{n=1}^{\infty} A_n \cos n\omega t \quad \dots\dots(3), \quad n$$

where

$$A_0 = \frac{\omega}{\pi} \int_0^{\pi/\omega} \sin^2 \left\{ \frac{\pi}{2} \left(\frac{b + a \cos \omega t}{V} \right)^2 \right\} dt \quad \dots\dots(4), \quad A_0$$

and

$$A_n = \frac{2\omega}{\pi} \int_0^{\pi/\omega} \sin^2 \left\{ \frac{\pi}{2} \left(\frac{b + a \cos \omega t}{V} \right)^2 \right\} \cos n\omega t \cdot dt \quad \dots\dots(5). \quad A_n$$

* W. D. Wright, *Proc. Phys. Soc.* **44**, 325 (1932).

The magnitudes of $A_1, A_2, A_3 \dots$ determine the nature and extent of the distortion and in particular the expression $100A_2/A_1$ (called "the percentage of second harmonic") is a convenient measure of the distortion. Thus in sound-recording it is usually reckoned that 5 per cent of second harmonic is the maximum distortion tolerable.

Now equation (5) may be written

$$A_n = \frac{1}{\pi} \int_{-\pi}^0 \cos(\alpha + \beta \cos \phi + \gamma \cos 2\phi) \cos n\phi \cdot d\phi \quad \dots\dots(6),$$

$$\begin{array}{lll} \phi, \alpha & \text{where} & \phi = \omega t \quad \alpha = (b^2 + \frac{1}{2}a^2) \pi/V^2 \\ \beta, \gamma & & \beta = 2ab\pi/V^2 \text{ and } \gamma = a^2\pi/2V^2 \end{array} \quad \dots\dots(7).$$

It can easily be proved that*

$$\exp\{\frac{1}{2}x(y - y^{-1})\} = \sum_{m=-\infty}^{\infty} y^m J_m(x) \quad \dots\dots(8),$$

J_m where $J_m(x)$ is a Bessel coefficient of order m . So that putting $y = i \exp(i\theta)$ and using the relation†

$$J_{-m}(x) = (-1)^m J_m(x) \quad \dots\dots(9),$$

we obtain

$$\exp(ix \cos \theta) = J_0(x) + 2 \sum_{m=1}^{\infty} i^m J_m(x) \cos m\theta \quad \dots\dots(10).$$

Therefore A_n is the real part of

$$\begin{aligned} \mu, \lambda \quad & \frac{\exp(ix)}{\pi} \int_{-\pi}^0 \left\{ J_0(\beta) + 2 \sum_{\mu=1}^{\infty} i^{\mu} J_{\mu}(\beta) \cos \mu\phi \right\} \cdot \left\{ J_0(\gamma) + 2 \sum_{\lambda=1}^{\infty} i^{\lambda} J_{\lambda}(\gamma) \cos 2\lambda\phi \right\} \cos n\phi \cdot d\phi \\ & = \exp(ix) \left[J_0(\beta) J_0(\gamma) \int_{-\pi}^0 \frac{1}{\pi} \cos n\phi \cdot d\phi \right. \\ & \quad + J_0(\gamma) \sum_{\mu=1}^{\infty} i^{\mu} J_{\mu}(\beta) \int_{-\pi}^0 \frac{2}{\pi} \cos \mu\phi \cos n\phi \cdot d\phi \\ & \quad + J_0(\beta) \sum_{\lambda=1}^{\infty} i^{\lambda} J_{\lambda}(\gamma) \int_{-\pi}^0 \frac{2}{\pi} \cos 2\lambda\phi \cos n\phi \cdot d\phi \\ & \quad \left. + \sum_{\mu=1}^{\infty} \sum_{\lambda=1}^{\infty} i^{\mu+\lambda} J_{\mu}(\beta) J_{\lambda}(\gamma) \int_{-\pi}^0 \frac{4}{\pi} \cos 2\lambda\phi \cos \mu\phi \cos n\phi \cdot d\phi \right] \quad \dots\dots(11). \end{aligned}$$

Of the integrals in this expression the first is zero; the second is (-1) when $\mu = n$, otherwise it is zero; the third is (-1) when $2\lambda = n$, otherwise it is zero; and the last is (-1) each time any of the following conditions are fulfilled:

$$2\lambda + \mu = n, \quad 2\lambda + n = \mu \quad \text{and} \quad \mu + n = 2\lambda.$$

Otherwise it is zero.

With these conditions it is a simple matter to pick out an expression for A_n for any particular value of n .

* Watson, *Theory of Bessel Functions*, p. 14.

† *Ibid.* p. 15.

Thus:

$$A_1 = \cos \alpha [J_1(\gamma) \{J_1(\beta) - J_3(\beta)\} - J_3(\gamma) \{J_5(\beta) - J_7(\beta)\} + J_5(\gamma) \{J_9(\beta) - J_{11}(\beta)\} \dots] \quad A_1$$

$$+ \sin \alpha [J_0(\gamma) J_1(\beta) + J_2(\gamma) \{J_3(\beta) - J_5(\beta)\} - J_4(\gamma) \{J_7(\beta) - J_9(\beta)\} \dots], \quad A_2$$

$$A_2 = \cos \alpha [J_0(\gamma) J_2(\beta) - J_2(\gamma) \{J_2(\beta) + J_6(\beta)\} + J_4(\gamma) \{J_6(\beta) + J_{10}(\beta)\} \dots] \\ + \sin \alpha [J_1(\gamma) \{J_0(\beta) + J_4(\beta)\} - J_3(\gamma) \{J_4(\beta) + J_8(\beta)\} + J_5(\gamma) \{J_8(\beta) + J_{12}(\beta)\} \dots],$$

$$A_3 = \cos \alpha [J_1(\gamma) \{J_1(\beta) + J_5(\beta)\} + J_3(\gamma) \{J_3(\beta) - J_9(\beta)\} - J_5(\gamma) \{J_7(\beta) - J_{13}(\beta)\} \dots] \\ - \sin \alpha [J_0(\gamma) J_3(\beta) + J_2(\gamma) \{J_1(\beta) - J_7(\beta)\} - J_4(\gamma) \{J_5(\beta) - J_{11}(\beta)\} + \dots], \quad A_3$$

and so on.

Now in any practical use of these series the maximum value of γ is about 0.25 and that of β is about 2, so that the series converge very rapidly, two terms of each being sufficient to give their sums to about four figures. Bessel coefficients are tabulated at the end of Watson's *Theory of Bessel Functions* and have been plotted by Hague.*

The value of A_0 is comparatively unimportant, but it may be found by an analysis very similar to the above.

§ 3. RESULTS

Figures 1 and 2 show the percentage of second harmonic for various values of the amplitude of the applied voltage and the bias. In considering these curves it must be remembered that the percentage of second harmonic is not the only factor to be considered; for if the cell is used over a range in which the characteristic is

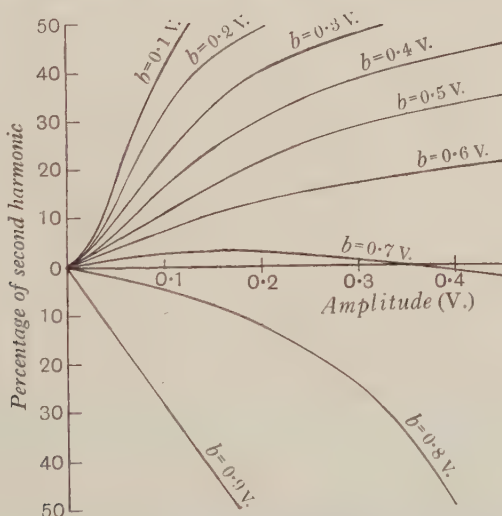


Figure 1. Percentage of second harmonic plotted against amplitude of the applied voltage at various bias voltages.

* *Proc. Phys. Soc.* 29, 211 (1917).

nearly symmetrical about the centre point the even harmonics will be small compared with the odd harmonics. For this reason an additional curve, figure 3, showing the relation between the percentage of third harmonic and the amplitude of the applied voltage when the biasing voltage is 0.7 is given.

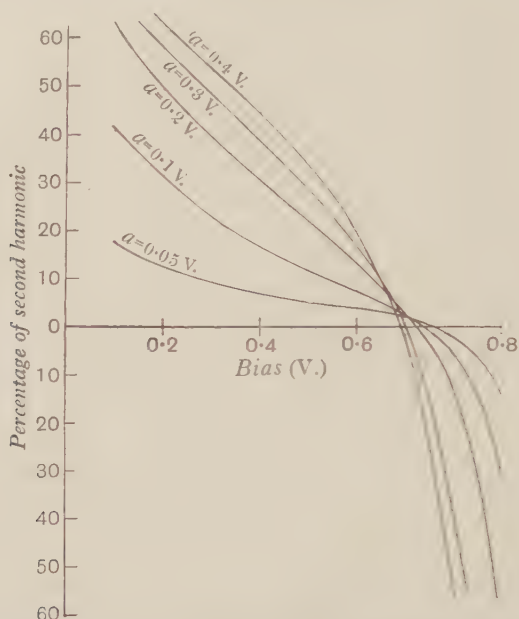


Fig. 2. Percentage of second harmonic plotted against bias at various values of the amplitude of the applied voltage.

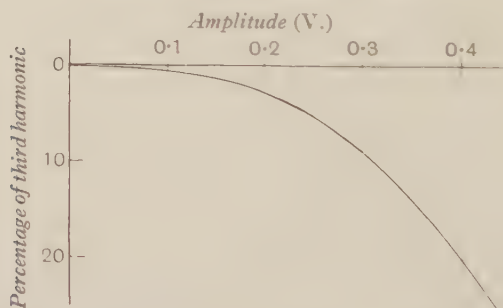


Fig. 3. Percentage of third harmonic plotted against amplitude of the applied voltage at a bias voltage of 0.7 V.

These curves support the conclusions of W. D. Wright* that for sound-recording a biasing voltage of about 0.7 and a maximum amplitude of just over 0.1 V. should be used. In television the need for large contrast has to be satisfied at the price of distortion, so that a biasing voltage of about 0.6 and amplitudes up to 0.4 V. are used, giving about 20 per cent of second harmonic and 9 per cent of third harmonic. Fortunately the eye is not very sensitive to this shade-distortion.

* *Loc. cit.*

DISCUSSION

Prof. A. O. RANKINE referred to the author's statement on page 472 that W. D. Wright had found the optimum biasing voltage to be 0.7, and asked whether this result was obtained by trial, or by calculation from equation (1).

Dr W. D. WRIGHT. I am afraid I cannot claim the credit that has been attributed to me for the values of the biasing and modulating voltages used in sound-recording. They originated in a paper by Zworykin, Lynn and Hanna* to which I referred in my paper on the Kerr cell and they were based on the same criterion as that used by the author of the present paper, namely that not more than 5 per cent of second harmonic is tolerable. The agreement found is therefore to be expected.

AUTHOR'S reply. I imagine that the conditions under discussion were initially arrived at merely by noticing that the characteristic curve is substantially straight between 0.6 V. and 0.8 V.

* *Trans. Soc. Motion Picture Engineers*, 12, 748 (1928).

AN EXPERIMENT BEARING ON TALBOT'S BANDS

BY THE LATE A. CHRISTOPHER G. BEACH, B.Sc., A.Inst.P.,
Chelsea Polytechnic

Received November 19, 1931. Read in title March 17, 1933.*

ABSTRACT. The author discusses the form of diffraction pattern observed in a spectroscope when a point of light is produced on the slit of the collimator, and an aperture in which the upper or lower half is covered with a retarding plate is between the collimator and telescope. He uses the bands so produced as a means to explain Talbot's bands, and shows how the method helps to find the best aperture-width to exhibit them. A description of the experimental methods and photographs of the effect are given.

§ 1. INTRODUCTION

TALBOT'S bands are produced in a spectroscope if white light is sent through it and half the beam is made to pass through a retarding plate of some transparent material, provided that the edge of the retarding plate is parallel to the slit and that the plate is inserted from the red side of the spectrum if placed between the collimator lens and the telescope objective, or from the other side if placed at the eyepiece. In order to get sharp bands there is a definite relation between the angular dispersion of the prism, the rate of change of phase-retardation produced by the plate, and the width of beam. The bands were discovered by Fox Talbot in 1837 and were explained by Airy, and later by Stokes. These two treatments are mathematical and are based on Fourier's analysis. In 1904 Schuster suggested an explanation based on the pulse theory of white light and, as a result, Talbot's bands have been considered to prove the pulse theory. Another method of explanation, which does not require the pulse theory, is that given by A. W. Porter†, who compares the arrangement to a two-aperture echelon grating.

During the summer of 1928 the author carried out some experiments the result of which also was to show that the pulse theory was not necessary to explain these bands. The method adopted, and shown in figure 2, was to focus the image of an electric-lamp filament L on the slit of a spectroscope and so to produce a point source of light at the focus of the collimator lens. When this point is viewed through the spectroscope the continuous spectrum is seen reduced to a horizontal line. If now the retarding plate F and aperture are placed as for Talbot's bands but rotated in their plane through 90° , a diffraction pattern is seen of oblique lines crossing the horizontal axis of symmetry and these oblique lines are cut into short lengths by dark bands nearly parallel to the axis.

* The Council regret the delay in publication of this paper, occasioned by the sudden and lamented death of the author. The paper in its original form was part of a thesis for B.Sc. by research.

† Preston, *Theory of Light*. Edited by A. W. Porter, p. 320 (5th edition).

Any point on this pattern can be considered as being produced by two displacements, one in the plane of the deviation, due to the spectroscope, and the other perpendicular to this, due to the diffraction pattern. The rate of change of these displacements with change of wave-length depends in the first case on the constants of the prism or grating, in the second on the rate of change of phase-retardation produced by the film and on the aperture-width, so that the slope of a line depends on the relative values of these two rates of change. If now the aperture is given such a width that the lines are at 45° , the two rates of change are equal and the aperture is the best to show Talbot's bands.

A series of points can be taken along the axis of symmetry of the spectra which are in the same condition of intensity of illumination. (As will be shown in the theory it is best to choose the points of zero illumination, as the zero-illumination lines are straight.) Let these points be considered as zero-displacement points, then on one side of each there is positive displacement for both axes and on the other there is negative displacement. Now turn the aperture and film through 90° in the plane of the film. This brings both sets of displacements into one axis. If the film is turned one way the displacements oppose each other and therefore sum to zero. This gives Talbot's bands. If the film is turned the other way they supplement each other and no bands can be produced.



Figure 1.

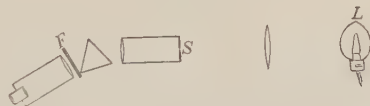


Figure 2.

On looking at the literature on the subject, the author found that N. K. Sethi had published in 1920* a description of somewhat similar oblique bands produced by a different optical arrangement. His method, figure 1, was to project the image of a slit *E* on to the slit of the spectroscope and then to place the aperture and retarding plate *F* close to the projection lens. The resulting effect at the slit of the spectroscope was analysed by the spectroscope and gave a similar diffraction pattern. The author has continued his experiments and now publishes them because his arrangement has advantages over Sethi's in that the development from the oblique bands to Talbot's bands can be carried out in a more logical and straightforward manner and, further, the method gives a means of determining quickly the best aperture-width.

§ 2. THEORY

In the mathematical discussion which follows the theory of the oblique bands is first considered, as these bands have an interest of their own. The results are then developed to show their bearing on Talbot's bands. The discussion depends on the theory of a rectangular aperture, and the usual textbook proofs are assumed but are developed further or by different methods where such a course is helpful.

* *Phys. Rev.* **16**, 519-525 (1920).

In each method an expression is obtained for the amplitude of the oscillations in a given direction. To change this expression into intensity it must be squared.

Let monochromatic light of wave-length λ pass normally through a rectangular aperture of sides e, e' , in the XY plane, figure 4.

In order to find the amplitude a of the oscillations in a direction OP let θ be the angle which OP makes with its projection on the YZ plane and θ' be the angle which this projection makes with the OZ axis. Then by the usual methods

$$a = A \frac{\sin \Phi}{\Phi} \cdot \frac{\sin \Phi'}{\Phi'} \quad \dots\dots(1),$$

where Φ, Φ' are phase differences and are of the form

$$2\Phi = \frac{2\pi e \sin \theta}{\lambda} \quad \dots\dots(2).$$



Figure 3.



Figure 4.

We shall only have to consider the variation in amplitude and intensity with variation of direction in one plane, say the XZ plane, and therefore Φ' and θ' can be considered fixed while Φ and θ are variables. If, in addition, our aperture is a long narrow aperture or slit and our plane is perpendicular to this and passes through its centre,

$$a = A \frac{\sin \Phi}{\Phi} \quad \dots\dots(3),$$

where A is the amplitude in the direction OZ .

The variation of a with Φ can be shown to pass through maximum, zero and minimum values by differentiating equation (3), but it is of interest to discuss this by means of a vector diagram, as such a diagram can be used to develop the theory of a long aperture in which half the aperture is covered by a transparent plate with its edge parallel to the long side of the aperture. Let the XZ plane cut the centre of the aperture. To find the effect in a direction OP in this plane, divide up the width e of the aperture into strip elements parallel to the Y axis. The amplitude of the disturbance in the given direction is the same for each strip, but the phase changes gradually in passing from one edge of the aperture to the other. Let this change be 2Φ . Then

$$2\Phi = \frac{2\pi e \sin \theta}{\lambda} \quad \dots\dots(2).$$

To add these disturbances by means of a vector diagram, let a vector of magnitude m represent the disturbances received from each strip of the aperture and be drawn

as a radius of m units within an angle AOB equal to 2Φ in a circle with centre O , as in figure 5. Let N be the total number of vectors. Then

$$Nm = A \quad \dots\dots(4), \quad N$$

and A is the effect in a direction P if all the disturbances arrive in the same phase.

If the vectors are not in the same phase but are spread over an angle 2Φ we combine the effects in pairs from two equal angles $d\phi$ at equal angles ϕ but on opposite sides of the bisector of AOB .

The number of vectors n within the angle $d\phi$ is $Nd\phi/2\Phi$ and their combined effect is $Nmd\phi/2\Phi$. The resultant along OR of the vectors contained in each pair of angles $d\phi$ is

$$\frac{2Nm}{2\Phi} \cos \phi d\phi,$$

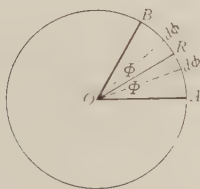


Figure 5.

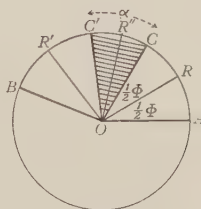


Figure 6.

and integrating over the range 2Φ we have

$$a = \int_0^\Phi \frac{Nm}{\Phi} \cos \phi d\phi,$$

$$a = \frac{Nm \sin \Phi}{\Phi},$$

or, writing A for Nm ,

$$a = \frac{A \sin \Phi}{\Phi} \quad \dots\dots(5).$$

It will be seen that Nm/Φ gets smaller at each cycle and therefore the maxima get smaller, and also that the resultant is zero each time OB completes a revolution—i.e. when $\Phi = n_1\pi$, where n_1 has the values 1, 2, 3, etc.

To apply this method to the case where half the aperture is covered by a transparent plate, let the plate increase the optical path by δ . If this increased path is expressed in wave-lengths of the particular light the path is increased by δ/λ wave-lengths, which may not be a whole number. In general there is thus a phase-difference of α between the light which just misses the edge and that which just passes through the plate.

Let figure 6 represent this. Each half of the vector diagram of the aperture has a maximum resultant of $\frac{1}{2}A$ when $\Phi = 0$, but the resultants are separated by an angle α . The final resultant is found by projecting R and R' on to a line which bisects the angle between them. This gives

$$a = 2 \left(\frac{1}{2}A \cos \frac{1}{2}\alpha \right),$$

and in general when Φ is not zero the angle between the resultants is $\alpha - \Phi$, and then

$$a = 2 \left(\frac{A}{2} \cdot \frac{\sin \frac{1}{2}\Phi}{\frac{1}{2}\Phi} \cos \frac{\alpha + \Phi}{2} \right) \dots\dots(6).$$

It will be seen that the resultant is zero in the vector diagram when the two component resultants oppose each other: i.e.

$$ROR' = \alpha + \Phi = \pi,$$

or when each of these component resultants is zero: i.e. when

$$AOC = 2\pi, C'OB = 2\pi.$$

In the first case

$$\cos \frac{1}{2}(\alpha + \Phi) = 0,$$

and in the second

$$\frac{\sin \frac{1}{2}\Phi}{\frac{1}{2}\Phi} = 0.$$

If the retardation is such that there is no phase-difference between the two halves, then the effect is the same as if no plate were present. If on the other hand $\alpha = \pi$, then when $\Phi = 0$ the amplitude is zero but it gradually increases with increasing Φ .

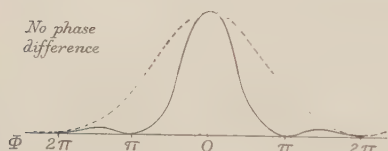


Figure 7.

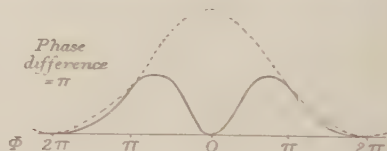


Figure 8.

These two conditions are shown in figures 7 and 8, where the intensity is plotted against the angle Φ . Figure 7 shows the condition when there is no phase-difference and figure 8 when there is a phase-difference of π between the two beams. The dotted curves are intensity curves for an aperture equal to half the given aperture, but the intensity ordinates are increased so as to make comparison easier. It will be seen that the maxima in figure 8 occur on each side between $\frac{1}{2}\pi$ and π . The positions in which they lie are found by differentiating equation (6) and then equating to zero. It is generally shown that $\Phi = \tan \Phi$ gives the position of the maxima for a simple aperture as in figure 7. Applying this method for the other case, figure 8, we have $\Phi = \tan \frac{1}{2}\Phi$.

In considering the case of white light, in order to make the discussion more precise let us suppose that white light radiates from a point and is then made parallel by a lens, after which it passes through a narrow horizontal aperture and is then spread out by a prism into a horizontal spectrum. This spectrum can be brought to a focus in some plane by a lens. There will be a diffraction pattern symmetrically arranged above and below a horizontal narrow continuous spectrum which stretches from red to violet; this line corresponds to the condition that $\theta = 0$. As there is a simple aperture the diffraction pattern consists of light and dark bands nearly parallel to the central band but slowly converging in the direction of the violet end,

because if λ is decreasing in equation (2) $\sin \theta$ must also decrease for any given value of Φ .

If now half the aperture be covered by a plate or film of thickness S_1 placed with its edge horizontal, there is a difference of optical path between the two halves of the beam amounting to $S_1 (\mu - 1)$; and expressing this difference in wave-lengths as before we have

$$N_1 = S_1 (\mu - 1) / \lambda \quad \dots\dots(7).$$

S_1

N_1

In this equation S_1 is a constant, $(\mu - 1)$ increases about 4 per cent in passing along the spectrum from red to violet, but λ becomes halved in the same range. The result is that the difference in phase passes rapidly through cycles from 0 to 2π as we pass along the spectrum. This is chiefly due to the variation of λ . The result is that the diffraction lines are tilted and cross the axis of symmetry for those wave-lengths for which $\alpha = 0$. In addition, the condition that $\sin \frac{1}{2}\Phi / \frac{1}{2}\Phi = 0$ must be taken into account. It will be seen that this is independent of λ and corresponds to the condition that the angles AOC and C_1OB are each equal to 2π in the vector diagram and therefore have no resultants. This condition gives dark bands corresponding to the 2nd, 4th, etc. bands of the plane aperture, and they run right along the system from end to end.

To apply these arguments to Talbot's bands: let a system of bands as described be produced by inserting a horizontal aperture and plate in a spectroscope as in figure 2. The width of the aperture and the thickness of the plate are so adjusted that the oblique bands are at 45° where they cross the central axis. Let θ be measured from a horizontal plane passing through the aperture, and let ψ be the deviation produced by the spectroscope; then for this condition

ψ

$$\frac{d\psi}{d\lambda} = \frac{d\theta}{d\lambda}.$$

Let figure 9 represent the condition about a point $O = \psi$ where a dark line crosses the axis; then any point S' is formed by displacing S through a distance SS' equal to its distance from O and of the same sign. If now the same aperture and plate are inserted as for Talbot's bands, each point S is displaced in the direction of O equal

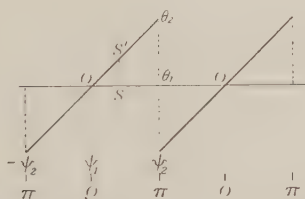


Figure 9.

to its distance from O and therefore every point on the oblique line from $-\psi_2$ to $+\psi_2$ is collected at O . If, on the other hand, the aperture and plate are inserted from the wrong side, all the points are displaced further apart over twice the distance from $-\psi_2$ to $+\psi_2$, and no bands are formed.

It will be seen that when the aperture and plate are in position to form Talbot's bands, the width of the beam through the spectroscope is limited to e and the centre of each dark band occurs where $z = \pi$, which is the distance on each side of ψ_1 at which the first minima occur. This fact has been recognized in all the theories of Talbot's bands but is very clearly shown by this method.

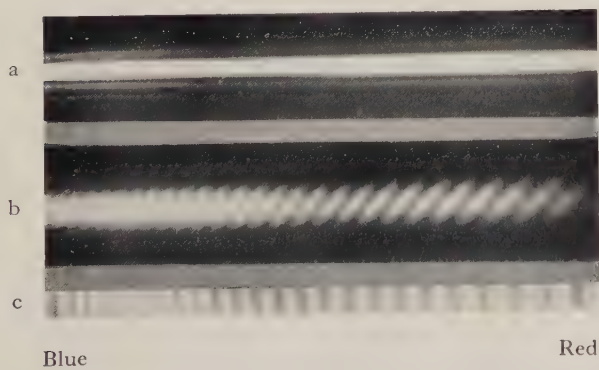
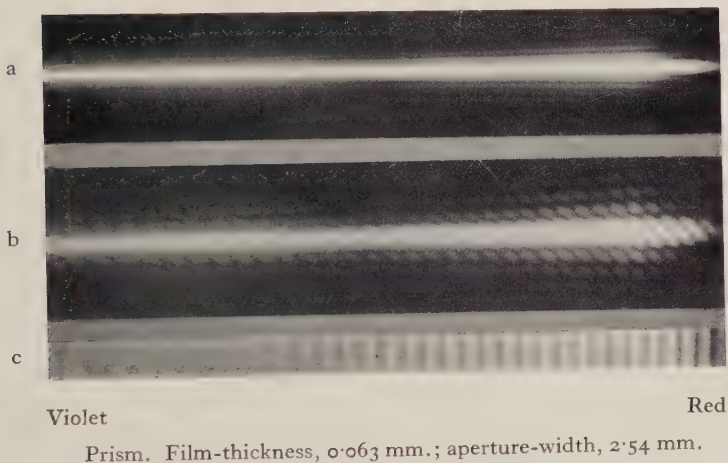
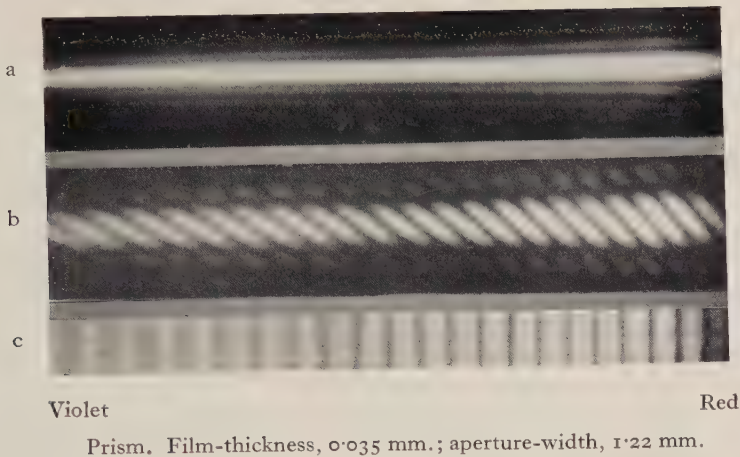
Although any series of repeating points can be taken as the series of origins O , the above argument most closely applies to the displacement and position of minima. As a result of the position of the maxima as shown in figures 7 and 8, the bright bands are not quite straight.

§ 3. EXPERIMENTAL

To produce the oblique bands described, and Talbot's bands, the writer used cellulose nitrate films. These films can be prepared of such a range of thicknesses that they give from 10 to 80 bands in the visible spectrum. The bands being broad are more easily seen than when a thicker material, such as a microscope cover-glass, which gives some hundreds of bands is used. To prepare these films cellulose nitrate varnish was diluted until it ran freely and was poured down a piece of plate glass standing almost vertical in a dish. The coat was allowed to dry and another was poured from a different edge. From two to eight coats were poured on in this way. When the last coat was dry the plate was placed in distilled water and the film floated off.

To show these effects visually apparatus was arranged as shown in figure 2. A spectroscope was set up with a glass prism in the position for minimum deviation. A 6-volt lamp L with a J filament having two straight limbs was focussed by an achromatic lens on to the spectroscope slit so that the image of one limb was perpendicular to the slit and crossed its centre. A simple brass slit in which one jaw could be slid out to give any aperture up to 1 cm. was clamped with its slit horizontal and close to the telescope objective. The transparent film or plate F was stuck across an opening in a card, and the card was fastened to the adjustable slit with plasticine, as it was necessary, whenever the aperture-width was altered, to move the film so that its edge should be along the centre of the aperture. Adjustment of the width was now made by trial and error until the bands occurred at 45° at their centre. This can only occur in one region of the spectrum at a time. The brass slit was now removed and measured and a permanent aperture of the same width was cut in thin fibre or card. Used with the same film this aperture gave the best bands in the part of the spectrum where the interference bands were nearest to 45° .

In order to photograph the effects it was found better to use the arrangement shown in figure 3. The slit S of the spectroscope was crossed by a second slit S' . The image of the sphere of a pointolite lamp was brought to a focus on the crossed slits so as to give a point source at the focus of the collimator lens. The adjustments were made as before. The eyepiece of the telescope was removed and the camera was placed in position. The camera consisted of a microscope objective mounted as in a photomicrographic apparatus, the objective taking the place of the telescope



a, uncovered aperture; b, horizontal aperture half-covered with film; c, Talbot's bands.

eyepiece. The advantage of this arrangement is that different magnifications can be obtained by changing the objective. The plate used was Ilford's Fast Soft-gradation Panchromatic plate, stated by the makers to have a speed of 2000 H. and D. A dilute methylene blue solution was used as a filter with the prism in order to reduce the intensity round the yellow part of the spectrum.

Two sets of photographs are shown taken with a 60° crown glass prism and one with a grating of 3600 lines to the inch. In each set is shown: (1) the diffraction effects of a simple horizontal aperture at F ; (2) the effect of covering half the aperture with an appropriate film; (3) the Talbot's bands when the aperture is turned to the correct position to show them.

§ 4. ACKNOWLEDGMENTS

The author's thanks are due to Dr Lownds for his interest in this research, and to Miss B. M. Pam, who made the varnish and films.

DISCUSSION

Dr J. R. MILNE. A simple and concise explanation of Talbot's Bands was given by the late Mr James Walker of Oxford. It was published in the *Philosophical Magazine* for April 1906, and also in the *Proceedings of the Physical Society*, volume 20.

DEMONSTRATION

“Experiments with ultra-short electric waves.” *Demonstration given on February 17, 1933, by N. L. YATES-FISH, M.A., D.Phil., Assistant-Lecturer in Physics at University College, London.*

The transmitter illustrated in figure 1 functions on the Barkhausen-Kurz principle* on a wave-length of 65 cm. The valve-holder projects through a hole in a vertical wooden board, and to the grid and plate sockets is connected a pair of horizontal wires 12 cm. long, forming with the internal leads of the valve an open-ended Lecher-wire system. The ends of these wires are attached to two vertical

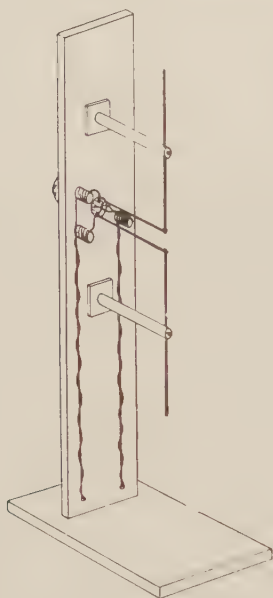


Figure 1.

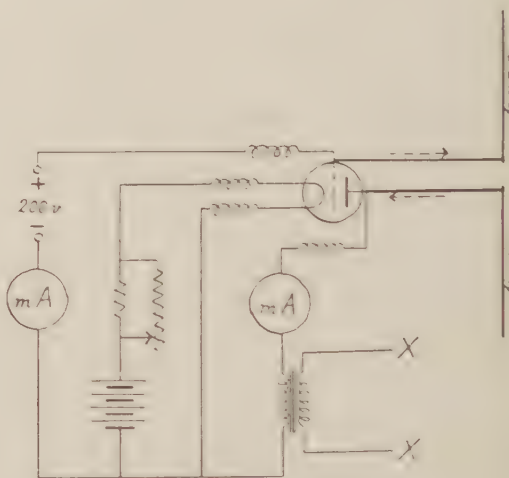


Figure 2.

rods 30 cm. (i.e. nearly half a wave-length) long, which form the radiating elements. In this way the radiation is greatly increased and tends to be concentrated in a horizontal direction. The circuit is given in figure 2, the arrows indicating the distribution of the oscillatory current. The high-frequency chokes at the points where the four leads are attached to the valve-holder have about ten turns each, the two filament leads being twisted together and wound on a single former.

The valve is a small army transmitting valve of type A T 40 taking a filament current of 1.5 A. at 7.5 V. The filament temperature is controlled by a rheostat

* See e.g. F. B. Pidduck, *Treatise on Electricity* (second edition), ch. xi; F. W. Chapman, *The Wireless Engineer*, 11, 108, September 1932.

of a few ohms' resistance shunted by a piece of resistance wire, this being the only critical adjustment. At the wave-length used the optimum grid-potential is about 200 V. (positive), the current lying between 100 and 200 mA. and the grid attaining a bright red heat. To indicate the setting-in of oscillations a milliammeter reading up to 20 mA. or so is inserted in the plate lead.

In order to produce an audible signal the transmitter is modulated by a low-frequency valve oscillator connected to the points *XX* in figure 2. This oscillator can if necessary be dispensed with, as it is found that if a low-frequency choke is connected in series with the plate lead the circuit will howl of its own accord at certain highly critical values of the filament current. Alternatively, unsmoothed direct-current mains can be used to supply the grid-potential.

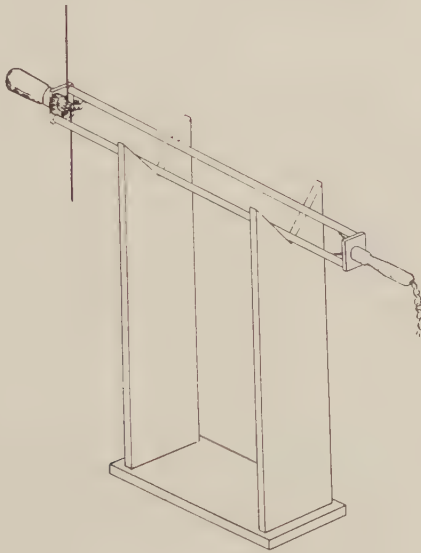


Figure 3.

The receiver, shown in figure 3, consists of an ordinary receiving valve (Osram HL 2) mounted at the end of a pair of wooden rods 80 cm. long and provided with a handle, the four leads being passed along the rods and through a hole drilled along the centre of the handle, thus ensuring a minimum of disturbance of the field around the antenna. Each lead is coiled into a small high-frequency choke of a few turns at the point where it is connected to the valve socket. The antenna wires, each 17 cm. long (the length is not very critical) are attached to the grid socket and one of the filament sockets respectively. The valve is biased as for anode-bend rectification and is coupled by means of a resistance in its anode circuit to an ordinary three-valve low-frequency amplifier and loudspeaker. The receiver can be rested on its wooden stand with the antenna either vertical or horizontal as desired.

The transmitter and receiver are set up several yards apart and as far as possible from all metal pipes, wires, etc. which would introduce stray reflections. Two screens, formed by stretching copper wires parallel to one another and 2 cm. apart across wooden frames 4 ft. square, are used in the experiments. One of these screens placed in the path of the waves with its wires vertical will almost completely cut off the radiation; with the wires horizontal, however, there is no effect, indicating that the waves are plane-polarized. This can also be shown by rotating the receiving antenna into a horizontal position, no signal being then received. If one of the screens is now interposed and rotated in its own plane a feeble signal reappears when the wires are at an angle in the neighbourhood of 45° to the horizontal; the screen is then causing a rotation of the plane of polarization. The reflection of waves can be demonstrated by cutting off the direct radiation with one screen and placing the other in a suitable position to reflect waves on to the receiver.

A very effective experiment is the illustration of the action of the Heaviside layer in producing fading of wireless signals. The layer is represented by a large metal sheet, or one of the screens with its wires vertical, held parallel to the plane containing the transmitting and receiving antennae. Interference takes place between the direct radiation and that reflected from the sheet, and by moving the latter to and fro the variations of intensity due to changes in the effective height of the layer can be simulated.

The existence of diffraction fringes is clearly demonstrated when the receiver is moved from side to side immediately behind one of the screens placed in the path of the waves.

The apparatus was constructed at the suggestion of Prof. E. N. da C. Andrade, who had planned to demonstrate the Heaviside layer by the method just described at his Royal Institution Lectures of November 1932 and required a suitable transmitter for the purpose. I have enjoyed throughout the benefit of frequent consultation with him.

REVIEWS OF BOOKS

Biographical Fragments, by Sir ARTHUR SCHUSTER, F.R.S. Pp. xiii + 268 and 8 plates. (London: Macmillan and Co., 1932.) 10s. 6d. net.

Readers of this book will be disposed to sympathize certainly with the feelings and possibly with the language of the archaeologist who unearths a few shards which shew they were once parts of a beautiful vase. For the early life of the author the fragments are fortunately fairly numerous, but for the later they are sadly deficient and can only be put together by the help of the indifferent clay provided by proceedings of scientific societies and catalogues of scientific papers.

Sir Arthur Schuster comes of a Jewish business family which a century ago had members in Frankfort-on-Main, Manchester and London. He was born in Frankfort on September 12, 1851. He succeeded when only 5, and still in petticoats, in getting sent to a private school with his elder brother. Here he was at first fairly happy and kept the top of his class, but an illness at 10 made it necessary for him to study under a private tutor in order to enter the gymnasium at 12. This tutor lived with the family for many years and helped Schuster in his mathematics when he left the gymnasium at 16½. An amusing account of the school is given, but he must have found the teaching uninspiring, had no desire to reach the top class, and left without regret. In the meantime Frankfort had been taken by Prussia during the seven weeks' war and the family had Prussian officers and men quartered on them. Anti-Prussian feeling ran high and Schuster's father decided to transfer his business and family to Manchester and to take on British nationality.

As Schuster and his two brothers were to enter the business, they were sent in turn to Geneva to improve their French. Schuster spent a year there and in addition to his French studies attended lectures in history, physics, chemistry, and anatomy, and did astronomical work in the observatory. This period seems to have been one of the happiest of his life and he records that he shed tears as the train left Geneva in May 1870 to take him to Frankfort, where he was to learn banking. The outbreak of the Franco-German war in July found him at Lytham, which had become the seaside resort of the family, and his business career began in the Manchester office of Schuster Brothers. By this time he had set his heart on a scientific career and the uncongenial office work was only partially compensated by attendance at Roscoe's evening classes in chemistry at the Owens College. His mother, who had now lost her sight, was the first to understand how unhappy he was, and it was due to her insight that his father consulted Roscoe, and that Schuster at 20 entered the college for a year to study physics under Balfour Stewart and mathematics under Barker. In a top room of the old Quay Street building he made the observations on the spectrum of nitrogen which formed the subject of his first paper, published by the Royal Society. After a further year at Heidelberg under Kirchhoff he obtained the Ph.D. degree, taking physics as his principal and mathematics and chemistry as his subsidiary subjects. His accounts of life and work in Heidelberg in 1872 are interesting and valuable. The story on p. 219 of the unruffled way in which Kirchhoff, after announcing at his Thursday morning lecture that he would be unable to lecture on Friday morning but would lecture as usual on Monday, used the Friday morning for his wedding and the week end for his honeymoon may have led to the legend of the professor who after setting his class to write an essay "while he attended to a little business," returned before the end of the hour and collected the scripts, having got married in the interval. On his return to Manchester in the summer of 1873 Schuster was appointed demonstrator in the physical laboratory of Owens College, which then consisted of three rooms in the basement of the

new building in Oxford Road. The following summer vacation he spent in Weber's somewhat primitive laboratory at Göttingen, and the autumn in Helmholtz's laboratory at Berlin, returning to Hampstead, where his father then lived, at Christmas 1874. He was at first asked to join, and ultimately to take charge of, the Royal Society expedition to Siam to observe the solar eclipse in April, 1875. An interesting account of the expedition occupies 40 or 50 pages, and is followed by one of a walking tour with two friends from Simla north over the Himalayas into Kashmir to the Indus at Leh, then west through Srinagar to Peshawar, a journey of over 600 miles which occupied two months. The 70 pages which the account of this journey occupies are full of graphic descriptions of the scenery, the natives, the difficulties from insubordination of the carriers, the rain, the bad roads, the cold and the scarcity of supplies. With 1875 the biographical details end and we have little more than a modern but equally disappointing version of the formula "the rest of the acts of — and all that he did, and his might, are they not written in the chronicles" of 1906?

The 20 pages of "Episodes" which follow contain some of the stories Schuster could tell so well. But there are many more. What of the glass prism which in the early days of spectrum analysis shewed an unusually generous separation of the sodium lines and was found afterwards to be quite impartial in its duplication of all lines? Or of the visitor to the laboratory in Manchester who claimed to be able to make rain at any time and at any spot he desired and offered (for a consideration) to give a practical demonstration? Or the potential contributor to the building fund for the new physical laboratories who demurred to giving to an institution in which Greek was taught, and the effect on him of Schuster's offer to guarantee that there should be no Greek further than a few letters of the alphabet used in the laboratory?

The 60 pages of "Biographical Byways" are reprints of Schuster's notes on physicists which have appeared at various times in the pages of *Nature*. Those dealing with persons known to the present writer seem to him to give a faithful and vivid picture of each one. Of Balfour Stewart as a lecturer the description on p. 206 is true. He had certain expressions which he used repeatedly, one of which was "thus you see." After a time his students would applaud each repetition, and when on occasions "gentlemen" was added the applause would be more than doubled. The difficulty of understanding Osborne Reynolds towards the end of his professorship, referred to on p. 232, were attributed by one who knew him well to his tendency "when speaking on any subject to be thinking of his *Sub-mechanics of the Universe* and when speaking of one part of the sub-mechanics to be thinking of another part."

The ten pages devoted to Dr Henry Wilde bring out clearly his intense appreciation of orderliness. In his train journeys in non-smoking carriages he invariably objected to any smoking. Although in verbal discussions no argument could move him at the time, a few days later he might remark "there is much to be said in favour of your point of view."

Two photographs of the author taken shortly before he left Manchester, and five of scenery in the Himalayas taken by Lady Schuster in 1908, add materially to the value of a book full of interesting reminiscences written in graphic and concise language, with humour but one serious fault—the "fragments" are not numerous enough. Some *Oliver Twist* should again be commissioned to go up to the master and say "please sir I want some more."

C. H. L.

Conduction of Electricity Through Gases, by Sir J. J. THOMSON and Prof. G. P. THOMSON. Pp. vi + 608. (Third edition, vol. 2. Cambridge University Press, 1933.) 30s.

This volume completes the third edition of the *Conduction*, vol. 1 having appeared in 1928. The first and second editions have long been out of print, and the completion of the third fills a distinct gap in the literature of the subject—a gap which perhaps appeared

widest to those who can remember the magnitude and direction of the impulse given to the development of experimental physics by the first appearance of this book.

The new edition is, naturally, much enlarged, and the author index (compiled by Mrs G. P. Thomson) to the second volume alone contains nearly 600 names. The greater part of this volume is given to recent developments, much of the older parts of the work and later refinements of the earlier measurements having been dealt with in the first volume. This second volume opens with a brief and largely familiar chapter on cathode rays, followed appropriately by a new section on the wave properties of these rays. There are long chapters on the collision of electrons with gas molecules and on ionization by positive rays and X-rays, still longer chapters on the complicated phenomena of the discharge in gases at low pressures and of the spark discharge, with shorter ones on reflected and secondary electrons from solid surfaces, on ionization by chemical and other actions, and on the electric arc.

It will be evident from this list of chapters that the authors have not been bound by a too literal interpretation of the title of the book; there are in fact large sections in which the processes of gaseous conduction are at most of secondary importance. The researches described, however, have developed so directly from the type of work which began in the detailed study of conduction in gases, and so largely in the hands of physicists of the same schools, that their inclusion in these volumes appears entirely natural.

It is a pleasure to find that the original character of the book remains so largely unimpaired, in spite of the expansion and rearrangement necessitated by the incorporation of so much new material. In its new as in its older forms, the *Conduction* is above all a book for experimental physicists, and one which is indispensable to research workers in many fields. For readers approaching the subject seriously for the first time it provides an authoritative introduction to many aspects of the most recent developments in atomic physics. New readers will particularly appreciate the wealth of relevant detail with which the more important experimental work is described. Further, as some space is found for accounts of the more significant work of the earlier investigators, the book contains in itself a fairly complete history of a great branch of electrical science. And the history of the thirty years following the first appearance of the *Conduction*—or the forty years since that of the *Recent Researches in Electricity and Magnetism*—is one which should be familiar, at least in outline, to all students of physics.

H. R. R.

Les Phénomènes Élémentaires de la Décharge Électrique dans les Gaz (Gaz rares), by Dr MARCEL LAPORTE. Pp. 232. (Paris: Les Presses Universitaires de France.) 75 fr.

This is one of the well-known series of "conférences-rapports de documentation sur la physique" of which a number have already appeared. It deals with the fundamental phenomena underlying the electric discharge with special reference to the inert gases, except that the methods of measuring critical and ionization potentials are for the most part omitted. These have been dealt with in an earlier book of the series by M. Bloch. Within the limitations of the subject chosen the author has given a very clear and satisfactory account of the present state of knowledge. He deals especially fully with the work of Townsend and his pupils, and devotes a good deal of space to considerations of elastic collisions between electrons and gas molecules. He does not, however, refer to the recent work of Arnot and others on the angular distribution of slow scattered electrons, which has thrown so much light on the earlier results of Ramsauer. Space is found for a discussion of the spectra of the inert gases, and tables are given of the terms and principal lines. In view of the importance, on many theories of the discharge, of the electrons ejected from solids

by the impact of positive ions more attention might perhaps have been given to an account of experiments on these lines, particularly those of Oliphant.

A feature of special interest is the careful account of the influence of the metastable states of the inert gases on their optical and electrical properties, especially through the action of "collisions of the second kind."

G. P. T.

The Principles of Optics, by Prof. ARTHUR C. HARDY, M.A. and FRED H. PERRIN, S.M. Pp. xiii + 632. (London: McGraw-Hill Publishing Co., Ltd.) 36s.

It is claimed by the authors of this very practical and useful book that "it should provide a solid foundation for those who intend to select optics as a career, and at the same time it should furnish an adequate knowledge of the subject in a comprehensible form for those who intend to specialize in other branches of physics or engineering." Its scope is very wide. Starting with the general concepts of geometrical optics and the Gaussian theory, together with chapters on lens aberrations and the diffraction theory of resolving power, the book goes on to deal with the physical optics of radiation and practical sources of light. We then find chapters on the eye, photography, and visual and physical photometry, including colour. Four chapters are devoted to optical materials and the manufacturing processes involved in the production of instruments, and the final series of chapters is devoted to the principal optical instruments. The practical applications of interferometry and polarization are also discussed.

It has been judged advisable to give this summary of contents here because treatises on optics may apparently contain anything from a most detailed and intricate discussion of the design of optical instruments to a superficial gallop over their general principles. This book, while perhaps casting its net too wide for great thoroughness, is very practical in its treatment throughout, and contains many useful tables such as those of reflecting power, the sensitometric characteristics of photographic materials, and the like, which will certainly be handy to those using the book as a work of reference.

A first impression regarding the book is that the authors are most successful in the physical-optics and photometric sections. Prof. Hardy's well-known work on the applications of photo-electric cells will lead most readers to expect an excellent treatment of physical photometry, and they will not be disappointed. Opinions may differ, however, in regard to the scope of the geometrical-optics sections, and it may be doubted whether the "solid foundation" for those entering "optics as a career" has in reality been laid. If the word "optics" implies the studies of an ophthalmic optician the scope is ample, but the treatment is not nearly systematic enough for any serious student of optical instruments. It will not be sufficient for him to accept the optical sine theorem without proof, or to learn that "a mathematical expression for the resolving power of a prism can be readily derived." A great deal of modern physical theory is intellectually unsatisfying, like the pieces of a badly fitting puzzle. This is inevitable and we accept it as a necessary stage in the evolution of various subjects, but where intellectual satisfaction which might be given to the student is denied him we encourage loose habits of thinking, and tacitly consent to the view that the satisfaction of the mind is of less importance than the production of microscopes and binoculars. It must be admitted, however, that it is easier to offer this criticism than to write an optical text-book against which it could not be made.

The authors write accurately throughout from a thorough knowledge, and errors are hard to find. (One which might be noted is the incorrect labelling of the image surfaces in figure 47). There must be very few who could not read the book without widening their knowledge of the subject, and it can be thoroughly commended as a general work of reference.

In view of the contemporary discussion of optical signs and conventions, it may be of interest to note that the following statement of the sign convention is given (we quote in part only): (1) Draw all figures with the light incident on the reflecting or refracting surface from the left. (2) Consider the object distance s or PV positive when P is at the left of the vertex. (3) Consider the image distance s' or VP' positive when P is at the right of the vertex. The fundamental equation for refraction at a curved surface therefore appears in the form:

$$\frac{n'}{s'} + \frac{n}{s} = \frac{n' - n}{R}.$$

The two focal lengths of a system are given the same sign, but the dioptric power is not used in the systematic treatment of lens theory, which avoids the more complex questions such as those of combined reflecting and refracting systems.

L. C. M.

The Form and Properties of Crystals: an Introduction to the Study of Minerals and the use of the Petrological Microscope, by A. B. DALE, M.A. Pp. x + 186. (London: Cambridge University Press.) 6s.

This account of crystals, their properties, and the methods by which these are investigated, is worthy of commendation. It is evident that every effort has been made to state the subject-matter in the simplest possible way, the similes in particular being extremely well chosen. Examples of these are the Great Pyramid, a match-box and a book to illustrate questions of symmetry; and the appearance of a new-mown lawn is particularly suitable for giving an idea of repeated twinning.

One of the most useful sections is that dealing with stereographic projections. It is certainly the best account of a subject which gives much difficulty to students and which is either ignored or treated inadequately by most writers. Excellent also is the attempt to explain physical properties such as cleavage; even if the suggested explanation is not altogether convincing, it is something to have the problem recognized.

The illustrations are good and give evidence of much thought and careful preparation; it is therefore unfortunate that some of the crystal drawings are slightly out of position, as, for example, in the case of figure 28.

To some minor points exception may be taken. Talc does not seem to be quite at home with the amphiboles. If in Mohs' scale "each mineral is harder than the one immediately below it," it is surely desirable to place diamond at the top of the table and talc at the bottom. Rotating and revolving are not quite the same thing. There is some uncertainty regarding ordinates and abscissae.

However, even the sun has spots, and these small defects should not obscure the fact that the book will be definitely useful in the hands of an earnest student who uses it to supplement his studies of actual crystals, models, and thin sections.

H. G. S.

Hydrodynamics, by Sir HORACE LAMB, M.A., LL.D., Sc.D., F.R.S. Pp. xv + 738. (Sixth edition. London: Cambridge University Press.) 45s.

What was effectively the first edition of this work was published in 1879 and the fifth edition in 1924. Every student of the subject must feel a personal debt of gratitude to the author for the great work which he has done in giving the world this magnificent exposition of the mathematical theory of the motion of fluids, and every student will rejoice at the abundant evidence supplied by this last edition that the author's powers of exposition still flourish with undiminished clarity, judgment and vigour. The book is beautifully printed

and bound, and the present edition reproduces all the clear and useful diagrams of the fifth edition, with the exception of Froude's sketch of the surface waves due to the bows of a ship (§ 256). The reviewers of previous editions of this work have said everything that can be said in its praise, and all that can be usefully attempted by the present writer is a sketch of the new matter introduced in the present edition.

Probably the most important of the new sections are those which bear on problems of aeronautics. The now famous theorem of Kutta and Joukowski giving the "lift" on an aerofoil in terms of the circulation is proved in § 72 b by an elegant method due to Blasius and in § 370 b by another method. The approximate treatment of the "boundary layer" due to Prandtl, Blasius and Kármán is given in §§ 371 a, b. § 371 c gives a brief reference to the effect of turbulence and §§ 371 d, e, f, and g study the effect of compressibility. Some related researches by Glauert and G. I. Taylor on the forces on immersed solids are summarized in §§ 134 a and 143.

References to some recent work on the theory of tides are inserted in chapters 8 and 9. It is shown that Rayleigh's principle can be used in the approximate calculation of free modes of oscillation (§ 205 b). A summary is given of the work of Goldsbrough and Colborne on the dynamical theory of tides (§ 223 a), and of the work of Havelock, Green and Lamb on surface waves due to travelling disturbances (§ 256 a). § 311 a gives a further development of the theory of long waves in an atmosphere in isothermal or convective equilibrium, and § 360 reproduces Rayleigh's investigation of the influence of viscosity in sound waves of permanent type. There are a number of other minor additions, e.g. an appendix to chapter 5 gives the hydrodynamical equations in general orthogonal coordinates: §§ 72 a and 121 a show that the effect at a distance from a moving solid immersed in a fluid is approximately equivalent to a double source; § 166 a gives Bjerknes' circulation theorem under meteorological conditions when pressure and density are independent. But the chief additions are those relating to aeronautics and tides, which are summarized above.

This book is a worthy monument to the erudition and powers of exposition of the author. It gives an unrivalled exposition of hydrodynamics regarded as a branch of pure mathematics.

G. T.

The Method of Dimensions, by ALFRED W. PORTER, D.Sc., F.R.S. Pp. vii + 80, with 9 diagrams. (London: Methuen & Co., Ltd.) 2s. 6d.

This little book describes some of the most familiar applications of the "method of dimensions" or, as Rayleigh preferred to say, the use of the principle of similitude. After "Prolegomena" in chapter 1, the next five chapters relate to the flow of fluids, surface tension, vibrating systems, temperature and heat effects, and electricity and magnetism. The last of these chapters is very brief and omits the interesting recent use of the principle to generalize the Child-Langmuir equation. No reference is made in the text, nor in the bibliography, to the books on the same subject by Bridgeman, Wallot and N. R. Campbell.

The Observatories Year Book, 1930. M.O. 340. Pp. 443. (H.M.S.O., 1933.) £3.3s. 0d.

It is always satisfactory to a worker dependent on measurements made at one of the British observatories to know not only that he will find the results given in the year books in a convenient form, but also that there is a carefully written statement of the methods of measurement and of reduction. In how many foreign or dominion publications of upper-air results do we find any definite statement of the method of calibration of the hair hygrometer, or of the amount of lag in the readings of pressure and temperature? Another satisfactory feature is the evidence that this work is carried out by men active in research

and is not governed by slavish adherence to routine. Thus instead of finding that the height at which an upper-air temperature or pressure reading was recorded is given as 17 dynamic kilometres—a statement that requires considerable explanation to a physicist unfamiliar with the modern jargon—we are correctly told that the height above mean sea level was such that the geopotential there was 17 kiloleos, a kiloleo being 10^8 c.g.s. units of geopotential. Further evidence of the lively interest of the staff in their work is the amount of their contributions to knowledge: those published during the last five years have included papers on subjects as varied as ground temperatures, magnetic disturbances and sunspot changes, atmospheric rainfall distribution in homogeneous air currents and at surfaces of discontinuity, smoke particles and condensation nuclei, air waves from gunfire, and earthquakes. Among much interesting detail is an example of the way in which the purely scientific becomes the commercially important—the hourly values of magnetic declination of Eskdalemuir are published weekly, primarily for the use of mine surveyors, in *The Colliery Guardian* and *The Iron and Coal Trades Review*. The meteorological department is to be congratulated on this useful volume.

G. T. W.

Données Numériques de Spectroscopie: Spectres d'Émission, par L. BRUNINGHAUS; *Spectres d'Absorption*, par V. HENRI; *Électro-magnéto-optique*, par F. WOLFERS; *Diffusion de la Lumière*, par P. AUGER. Pp. xxi + 1397. (Extracted from vols. 8 (1927-8) and 9 (1929) of *Tables Annuelles de Constantes et Données Numériques*. Paris: Gauthier-Villars et Cie, 1932.) £3. 13s. od.

To the many English users of the *Annual Tables* the general features of the present volume will be well known, for it is a rebinding of the spectroscopic sections of two of the later volumes of the full work. This reissue is an excellent idea for which all spectroscopists and many other workers in chemistry and physics cannot but feel grateful to the International Committee and its enterprising general secretary, M. Ch. Marie. It renders a huge mass of collected spectroscopic data available to laboratories (and perhaps even to some individual workers) whose limitations of interests and financial resources preclude them from subscribing to the whole series of volumes of the *Annual Tables* just for the sake of the spectroscopic sections. The volume weighs nearly 8 pounds and contains some 1400 large pages ($10\frac{1}{2} \times 8\frac{1}{2}$ inches) of fairly closely printed matter in the form of notes and tables taken bodily from the original papers. The subjects to which the data relate include line spectra of neutral and ionized atoms, band spectra and Raman spectra of diatomic and polyatomic molecules, and the Zeeman and Stark effects. As is well known, the *Annual Tables* make no pretence to be critical, the fullest possible attention being paid to each paper no matter whether it is, on the one hand, one in which the data are good, the analysis is accurate and the notation universally recognized, or, on the other hand, one in which the data are only moderately accurate, the analysis (if any) is erroneous or incomplete, or the notation known to few besides the author himself. Indeed, in regard to the band spectra of diatomic molecules the volume has the bad luck to deal with papers which appeared before the chaotic state of the notation was resolved by the international adoption of Mulliken's notation of 1929 and 1930; this is inevitable since the printing of the 1927-29 volumes is used for this 1932 volume apparently without resetting. However, the uncritical and heterogeneous nature of the tables need not disturb anyone by whom they are used, as indeed they must be used, in conjunction with later accounts of the subjects concerned. Spectroscopists using, say, the volumes on line spectra and atomic energy states in the McGraw-Hill Company's International Series, or the reports on band spectra published by the American Physical Society and by our own Physical Society, will find the present volume of great value.

W. J.

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